Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms, of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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Plans and Hopes for Applied Mechanics Reviews

THE aims of a magazine can not be completely apparent from a single issue—especially a first issue which for many reasons is hardly typical. Hence a few words of explanation are in order. This magazine has been founded for the purpose of helping workers in Mechanics throughout the world to keep informed about the work which is being done everywhere in this field. It will be directed particularly toward research workers, but it should also be of value to many others—teachers, designers, etc.—who must use the *results of research* in Mechanics.

To serve the above purpose an attempt will be made to scan all sources in which interesting material may appear, and to select and review the papers presenting new developments, as well as books and other material of interest to research workers. The period from January, 1947, on will be covered. Due to the volume of present day literature and obvious limitations of space, a maximum of a few hundred words can be devoted to each item. It is impossible to present even the "meat" of many articles in such a space, but in all cases it is our aim to give enough information to enable the reader to decide whether he should read the original article.

Thus Applied Mechanics Reviews must of necessity be more a guide to reading than a source of detailed information, although of course the important ideas, facts, and conclusions in the papers reviewed will be presented when possible. However difficult to find or read a paper may be, every paper reviewed will, if it is important enough to a reader, be available to him through our photostat service and the commercial translating agencies.

Each review will be a "critical" review written by a specialist or authority in the particular field involved. An attempt will be made to level all national barriers not only in the material covered but in the selection of reviewers. Also, through the co-operation of outstanding authorities in this field and of engineering and scientific societies throughout the world, it is hoped eventually to make the direction of the policy of this magazine an international affair, even though any such undertaking must be centered in some particular country. This first issue is by necessity largely American, and because of difficulties in getting source magazines the coverage is somewhat "spotty." This situation can be expected to change.

The Editors will be grateful for criticisms and suggestions for increasing the usefulness of this magazine. All suggestions will be carefully considered although of course factors unknown to the senders may often prevent their being used. Among the many suggestions already received is the interesting one, proposed by M. G. Scherberg, of maintaining a "contact column" or "question and answer" section. This would print brief signed requests for information on specific research topics, transmit the replies received and publish those of general interest. Such a feature might facilitate an interchange of ideas between research workers on a different but no less valuable plane from that afforded by the reviews. Comments are solicited.

Finally, the Editors wish to thank all those who have helped and are helping in this undertaking. Because of the pressure of work it is impossible to individually acknowledge much of this assistance, but it is nonetheless appreciated. Particularly to be thanked are the reviewers, whose unselfish and oftentimes difficult work makes this magazine possible.

L. H. Donnell

General Theoretical and Experimental Methods

See also Revs. 12, 13, 18, 37, 39, 138, 139, 146, 159, 162, 179

1. L. Fox, "Some improvements in the use of relaxation methods for the solution of ordinary and partial differential equations," *Proc. roy. Soc., London, Ser. A*, June 17, 1947, vol. 190, no. 1020, pp. 31–59.

In the solution by numerical methods of ordinary and partial differential equations, the derivatives are replaced by their finite-difference equivalents, usually infinite series of differences. For the employment of relaxation methods it has been customary to ignore all but the dominant first term of these series, taking sufficient intervals to insure the effective vanishing of the neglected terms. Methods are developed in this paper whereby the full difference equations can be used at the maximum interval consistent with the convergence of the differences. The number of mesh points, and hence the labor, is thus reduced, and increased accuracy is obtained.

The general method of solution is as follows: The difference corrections are ignored, and a first approximation obtained by relaxation methods in the usual way. These values are then differenced, and, if the differences are obviously converging, the difference corrections are calculated and entered as a new residual, to be relaxed in turn. This is continued until the full finite-difference equations are satisfied. The actual details depend on the equation considered. Eight examples have been carried out, covering ordinary and partial differential equations, eigenvalue problems, and problems involving curved boundaries.

C. T. Wang, U.S.A.

2. R. H. Hughes and E. B. Wilson, Jr., "An electrical network for the solution of secular equations," *Rev. sci. Instrum.*, Feb. 1947, vol. 18, no. 2, pp. 103–108.

An electrical instrument for calculating the roots of secular or determinantal equations is described. The instrument is based on the correspondence between the resonant modes of an electrical network and the characteristic values of a square matrix.

A detailed description is given of a particular instrument, which solves symmetrical matrices with real elements which have been reduced to the form where the characteristic numbers appear

only in the main diagonal and are of the first degree in each element. Experience with this instrument has shown that the roots may be determined with an error of less than 2 per cent. Although not mentioned in the article, application of the method to asymmetric matrices, matrices with complex elements, and other forms of matrices will occur to the reader.

C. E. Warren, U.S.A.

3. Aurel Wintner, "A criterion for stable characteristic exponents," *Quart. appl. Math.*, July 1947, vol. 5, no. 2, pp. 232–236.

In this paper two mathematical tests are constructed for the stability (i.e., boundedness as $t \to \infty$) of the solutions of the equation $x'' + f(t)x = 0$, where $f(t)$ is periodic. This question might well arise in either structural or electrical oscillation problems.

The tests are carried out as follows: Let $M = \max|f(t)|$, $\mu = \int_0^1 f(t)dt$, where the period of f is unity. Then,

$$M^2\exp(M) < 6\ \mu \leqq 6, \text{ and}$$

$$M^2\exp(M) < 24 - 6\ \mu \leqq 12$$

each imply that the solutions are stable. The test is applied to an illustrative problem concerning frequency modulation.

G. F. Carrier, U.S.A.

4. W. P. Mason, "Measurement of the viscosity and shear elasticity of liquids by means of a torsionally vibrating crystal," *Trans. Amer. Soc. mech. Engrs.*, May 1947, vol. 69, no. 44, pp. 359–370.

Some measurements of the viscosity coefficient of liquids and gases are given, as obtained by an ingeneous experimental arrangement involving a torsionally oscillating cylindrical crystal. The viscous waves thus excited in the surrounding medium introduce a loading effect on the crystal, which can be measured by the increase in the resonant resistance and the decrease in the resonant frequency. The chief value of the paper lies in the exhaustive analysis of the method and the errors involved.

Some measurements of viscosity coefficients for light liquids, compared with those obtained by flow methods, show deviations as high as 11 per cent. These deviations are ascribed to the lack of temperature control in the apparatus used. Some measurements of the compressional viscosity coefficient λ agree with results obtained by ultrasonic absorption in indicating that the Stokes relation $\lambda = -(^2/_3)\eta$, where η is the coefficient of shearing viscosity, is in error an amount dependent upon the frequency.

A. M. Kuethe, U.S.A.

5. H. Motz, "The treatment of singularities of partial differential equations by relaxation methods," *Quart. appl. Math.*, Jan. 1947, vol. 4, pp. 371–377.

A method for obtaining solutions of certain partial differential equations with sharp corner boundaries is described. The necessity for using a fine mesh net in the vicinity of the sharp corner is overcome by introducing an approximate mathematical solution of the equation, which satisfies the boundary conditions in a region near the corner. The two-dimensional Laplace and wave equations are treated in the paper. The use of the method for removal of boundary singularities by conformal transformations is outlined. C. E. Warren, U.S.A.

General Dynamics, Kinematics, Friction

(See also Revs. 22, 50, 153, 173)

6. David E. Weiss, "Design and application of accelerometers," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 4, no. 2, pp. 89–99.

This paper treats of spring mass systems of one degree of freedom composing the seismic elements of electromechanical transducers for accelerometers, the parameters being mass, spring-rate, and viscous-damping coefficient. Conditions for steady-state response as an accelerometer or as a displacement pickup and an expression for dynamic sensitivity are given for various damping ratios. An increase of natural frequency is shown to result in reduced sensitivity.

Phase lag conditions for several damping ratios are given, and it is shown that when the damping coefficient is about 0.7 of critical, the phase lag is approximately linear up to resonance, so that an instrument reproduces a true acceleration curve. In the case of steady-state vibrations, this eliminates free vibration and gives dynamic sensitivity over a large frequency range with minimum phase distortion.

The more complicated problem of transient states is simplified by assuming rectangular and triangular step functions for the imposed accelerations. Solutions are given.

It is concluded that: The reading of an accelerometer is a function both of the magnitude of the applied acceleration and of a time parameter; under steady-state conditions, the natural frequency should be two or three times the highest applied frequency of interest, and the damping ratio 0.6 to 0.7; high natural frequencies are required to record transients but the natural period should be short, relative to the pulse duration. Preferred natural accelerometer frequencies for aircraft flight and landing tests are given. Several applications are discussed.

Lloyd R. Koenig, U.S.A.

7. L. Landweber and M. H. Protter, "The shape and tension of a light flexible cable in a uniform current," *J. appl. Mech.*, June 1947, vol. 14, no. 2, pp. 121–126.

This paper concerns the shape and tensions of a towing cable. In distinction to an earlier work by Glauert, both the normal and tangential components of the fluid force on cable elements are considered. The weight of the cable is neglected. The further assumption is made that the tangential component is constant. This was verified as a good approximation by Model Basin tests.

The equilibrium of a cable element yields, by direct integration, equations for tension and length of arc s as a function of the angle ϕ between a cable element and the direction of motion. Next, the co-ordinates of the arc in the form of integrals are obtained by the use of the parametric equations $dx/ds = \cos\phi$ and $dy/dz = \sin\phi$. Numerical evaluation of the integrals is greatly facilitated by the introduction of functions peculiar to the problem, of which an extensive table is appended. Four illustrative examples are given. F. Hymans, U.S.A.

Gyroscopics, Governors, Servomechanisms

(See also Revs. 13, 14, 15, 21)

8. Herbert Harris, Jr., "The analysis and design of servomechanisms," *Trans. Amer. Soc. mech. Engrs.*, Apr. 1947, vol. 69, no. 3, pp. 267–280.

This paper presents methods of analysis and design of the simpler types of servomechanisms, characterized either by no time delays in response, when the system degenerates to the ordinary vibrating system, or by one or two linear time delays. The author shows how positionally regulated systems, with their time delays expressed in the standard operational form of $(1 + T_1p)\ (1 + T_2p)$ etc., in the denominator, can be provided with derivative response which leads to terms $(1 + t_1p)\ (1 + t_2p)$ etc., in the numerator of the expression for output torque, and may thus be used effectively to cancel the effect of the time delays.

Curves based on differential analyzer solutions are given for system response to: (1) an initial output displacement error; (2) a suddenly applied constant input displacement; (3) a sud-

denly applied constant input velocity; and (4) a suddenly applied output load torque. A discussion of error-integral control as a means of reducing steady-state error is also given. Limits on the amounts of first and second derivative and error-integral signal to use are established by practical considerations of speed of response, presence of dry friction in output, type of input signal, etc., rather than by recourse to Routh's criteria of stability. This is a regrettable omission in an otherwise good summary of elementary theory. Stanley J. Mikina, U.S.A.

9. H. I. Tarpley, "An instrument to measure servomechanism performance," *Rev. sci. Instrum.*, Jan. 1947, vol. 18, no. 1, pp. 39–43.

Equipment is described for measuring the transient and steady-state transmission through a system designed to transmit an angular-position signal. This system may be of the open or closed-loop type.

For measuring transient angular-position response to an input test signal such as a step-function, the output shaft position is first converted to a voltage, for example, by a potentiometer. The response voltage is then sampled at equal time intervals by a set of condensers which are connected in sequence by a commutator. The recorded voltage samples are then measured individually by use of a cathode-ray oscilloscope.

Means are discussed for measuring the amplitude-frequency characteristic of an angular-position transmission system. Equipment is also described for measuring the phase-shift—frequency characteristic of such a system. Input and output angular positions of shafts are converted to electrical signals which are used to modulate a 400-cps carrier signal in the phase-shift measurement. John L. Barnes, U.S.A.

10. R. A. Brown, "Latest developments in aircraft controls and instrumentation," *Trans. Amer. Soc. mech. Engrs.*, Feb. 1947, vol. 69, no. 2, pp. 109–116.

The paper outlines broad principles of electronic control systems having Wheatstone and similar bridge circuits, particularly those employing potentiometer balancing sections. It outlines remote positioning means for practical applications wherein full sensitivity with minimum hunt is desired. Means are specified for attaining sensitive initial responses of a servomotor with dead-beat performance.

Applications to automatic pilots, turbosupercharger regulators, engine temperature controls, capacitance-type fuel gages, and cabin temperature controls are discussed. Pertinent circuit diagrams are included. Advantages of circuit-soldered tubes, hermetically sealed transformers, block constructions of resistances and capacitances, and flat-ribbon potentiometers are mentioned. Lloyd R. Koenig, U.S.A.

11. F. Hymans, "Centrifugal governor in elevator service," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, no. 8, pp. 881–890.

This is a direct mathematical analysis of the performance of elevator safety governors during free fall. It is shown that the equation of motion of a representative spring-biased centrifugal governor is that of a vibrating system of one degree of freedom acted upon by a constant force and having an elasticity that is a function of time. Criteria of governor unbalance are deduced for free-fall conditions without slip between the governor rope and sheave. It is noted that in the ordinary adjustment the governor is slowly brought up to tripping speed under conditions of substantial equilibrium, whereas there is no equilibrium during free fall, so the actual performance of the governor may differ from that often assumed.

Several types of safety controls are discussed, including those responsive to excessive acceleration with safety application shortly after emergency, and those responsive to overspeed to bring about a safety application longer after emergency. Several examples of governor performance are worked out in detail with and without slip between the governor rope and sheave. The effects of reduced governor acceleration due to slip are indicated.

The analysis brings to light important factors which should be considered in rationally designing governor parts. The practice of adapting a given governor to different car speeds merely by change of gear ratio, and the maximum torque that may be transmitted by roping to the governor sheave under given traction relations are analyzed. The paper is illustrated by performance graphs, tables, and informative skeleton diagrams of the mechanisms involved. Lloyd R. Koenig, U.S.A.

Vibrations, Balancing

(See also Revs. 3, 4, 6, 8, 28, 79, 157)

12. George Polya, "On the fundamental frequency of vibrating membranes and the torsional resistance of elastic rods (Sur la fréquence fondamentale des membranes vibrantes et la résistance élastique des tiges à la torsion)," *C. R. Acad. Sci., Paris*, July–Dec. 1947, vol. 225, pp. 346–348.

An analogy is shown between the square of the fundamental frequency of a vibrating membrane which is fixed along a closed curve, and the reciprocal of the torque producing a certain angle change in a shaft having a cross section defined by the same closed curve. Useful generalizations concerning the relative effectiveness of various shapes may be developed by applying the symmetrization procedure of Steiner in conjunction with the analogy. Glenn Murphy, U.S.A.

13. N. Minorsky, "Experiments with activated tanks," *Trans. Amer. Soc. mech. Engrs.*, Oct. 1947, vol. 69, no. 7, pp. 735–747.

Ships may be stabilized against roll by any one of the following means: gyroscopes, antirolling fins, moving weights, and interconnected wing tanks with movable fluid. Further, any one of these devices can be utilized either as a passive or active roll suppressor. Reviewing these various devices the author points out the superiority of the activated tank arrangement.

Experimental work on activated tanks was initiated by the U. S. Navy Department, under the direction of the author, several years before the war. In these tanks the water is forced from side to side by a variable-pitch axial-flow propeller-type pump. Control is initiated by an angular accelerometer, activating a servomotor system, which in turn adjusts the pitch of the impeller blades.

The paper presents the theoretical basis of control of such a system and the experimental model work carried out to confirm the theory. The paper discusses the full-scale tests made on the destroyer *Hamilton*. The outbreak of the war made it necessary to discontinue these tests, but it is noted that activated tanks remain as perhaps the most promising of the antiroll devices.
Frank M. Lewis, U.S.A.

14. A. Kleiner, "Amplitude of oscillation and variation of speed in reciprocating engines (Pendelwinkel und drehzahlschwankungen der kolbenmaschinen)," *Schweiz. Bauztg.*, Aug. 16, 1947, vol. 65, no. 33, pp. 443–448.

The author advocates that codes for permitted nonuniformity in speed of engine generator sets be based on calculations which take into account the elasticity of the shaft. Such codes allow wider specified limits of torsional oscillations, since the calculations approach closer to the actual performance.

Specifications should be such that noticeable flickering of light is prevented, and that alternators can be operated in parallel.

The author argues that the frequently used quantity $\delta = (\omega_{max} - \omega_{min})/\omega$ (ω = angular velocity of shaft) is really not a good criterion for this performance. The value of δ at which flickering is noticeable depends on the flicker frequency, the eye being most sensitive to a flicker frequency of 7 cps. The paralleling of alternators depends on the electrical phase angle, both characteristics not fully specified by δ.

The equations for the torsional oscillations of a crankshaft with a number of sections are set up. A determinant is given for the calculation of the oscillation angle produced by each excitation frequency. The discrepancy between the oscillation amplitudes calculated under the assumptions of rigid shaft and flexible shaft is demonstrated for frequencies close to resonance.

Robert Fehr, U.S.A.

15. John L. Bogdanoff, "A method for simplifying the calculations of the natural frequencies for a system consisting of n rigid rotating disks mounted on an elastic shaft," *J. aero. Sci.*, Jan. 1947, vol. 14, pp. 5–18.

The problem of determining the forward and backward (resonant) precessional speeds for a system composed of a series of rotating disks mounted on an elastic shaft is treated in the same general manner as by previous investigators. A linear transformation is introduced in the equations of motion; this is a calculational convenience when the resonant speeds over a wide range of shaft speeds are of interest.

Martin Goland, U.S.A.

16. G. Krall, "Dynamics and aerodynamics of wires—visible vibrations (Dinamica ed aerodinamica dei fili. Premess. Vibrazioni visibili)," *R. C. Accad. Lincei*, July–Aug. 1947, ser. 8, vol. 3, 2nd sem., pp. 11–17.

Vibration phenomena of wires, in the field of electric power transmission, fall into two categories: (1) Visible oscillations of large amplitude, involving the hazard of two wires coming sufficiently close to produce short circuit. (2) Invisible, acoustic vibrations produced by wind forces, which endanger the strength of the supporting elements. This paper deals with the first category.

Previous treatments of large oscillations of wires, by Bernouilli, d'Alembert, Lagrange, Routh, and Wolf treated these phenomena theoretically and depend for their usefulness on the correctness of the basic assumptions. This paper endeavors to find practical answers to the problem formulated as follows: If, for a catenary of chord L, deflection f, cable radius r, and unit weight μ_0, a certain spacing distance has been found sufficient in practice, then what should be the proper spacing distance for another set of conditions: L', f', r', μ_0'?

The author develops the fundamental equation of vibration of a stressed cord under a disturbing force which is a function of time and location. Based on this general equation he investigates in detail, mathematically, the cases when: (1) The disturbance is periodic, and (2) a single disturbance occurs, as by a sudden detachment of an ice load. The mathematical derivations and final equations are supplemented by a numerical example.

K. J. De Juhasz, U.S.A.

17. G. Krall, "Dynamics and aerodynamics of wires—acoustic vibrations (Dinamica ed aerodinamica dei fili. Vibrazioni acustiche)," *R. C. Accad. Lincei*, July–Aug. 1947, ser. 8, vol. 3, 2nd sem., pp. 17–22.

This treats invisible, acoustic vibrations of a wire under tension, induced by a wind of constant velocity, and in particular the frequency of the resulting note. The results are applicable also to cylinders such as telegraph poles, and industrial smoke stacks. The treatment is based on the vortex streets discovered by Bénard and further developed by Kármán. The theory envisages two types of vortices generated by a body acted upon by a constant fluid current: (1) Vortices disposed symmetrically to the direction of motion, an unstable condition, and (2) vortices disposed alternately in zigzag formation, a stable condition.

The present treatment deals only with the latter, stable kind of vortex street. The Kármán theory determines the translational velocity of the vortices and their lateral and longitudinal disposition, findings which have been further developed by Krueger and Lauth. The paper gives an extensive mathematical derivation, and formulas are derived for the condition of resonance, and the maximum amplitude. It has been found that the amplitude of vibrations is independent of the length of chord. A numerical example is given.

K. J. De Juhasz, U.S.A.

18. G. D. McCann and J. M. Kopper, "Generalized vibration analysis by means of the mechanical transients analyzer," *J. appl. Mech.*, June 1947, vol. 14, no. 2, pp. 127–134.

This paper first presents curves showing the steady-state response of a linear vibrating system consisting of two masses and two springs (with damping) over a range of system parameters. One spring K_1 connects the two masses M_1 and M_2; the other K_2 connects one of the masses M_2 to a fixed point. The resulting force in the spring K_1 connecting the two masses is given for an exciting force applied at the mass M_2. The ratio M_1/M_2 varies from 0.02 to 1.0, and the ratio ω_1/ω_2 from 1 to 10, where $\omega_1 = \sqrt{K_1/M_1}$ and $\omega_2 = \sqrt{K_2/M_2}$.

The paper then gives curves showing the response of the mass M_1 (that is, the maximum force in the spring K_1) to an exciting force arising from the previously "fixed" point of the system (carrying the whole system along with it) striking a fixed base with a known velocity, with and without rebound. In this part of the study the mass M_1 and spring K_1 are assumed to be much smaller than M_2 and K_2, respectively, and the reaction of system 1 on system 2 is consequently neglected. As a guide to the application of the curves, a criterion for the occurrence of rebound is given in terms of the relative magnitude of a third mass M_3, which for this purpose replaces the fixed point.

Charles Concordia, U.S.A.

19. Charles E. Crede, "Damping of textile-mill movement by Frahm system," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, no. 8, pp. 937–946.

This article describes and gives the theory of tuned Frahm vibration dampers of considerable mass, which are installed on the upper floors of a building containing a large number of weaving looms. These looms inherently produce large alternating inertia forces in a horizontal plane. J. P. Den Hartog, U.S.A.

20. I. Epstein, "The motion of a conical coil spring," *J. appl. Phys.*, Apr. 1947, vol. 18, pp. 368–374.

The author derives the equation of motion of a conical coil spring and presents the solution in terms of Bessel functions of the space variable. Natural frequencies of the system are determined and the calculated results are checked by experiment for two springs, one of nickel and one of piano wire.

Albert I. Bellin, U.S.A.

21. Seymour Sherman, "A note on stability calculations and time lag," *Quart. appl. Math.*, Apr. 1947, vol. 5, pp. 92–97.

The stability of certain systems with time lag leads to the condition that the roots of the equation $az^2 + bz + \beta ze^{-z} + c = 0$ (where a, b, c, β are constants) have a negative real part. After reviewing the work of Minorsky and others, the author concludes that the method based upon replacement of e^{-z} by a finite number of terms of its expansion in powers of z may lead to errors, and has led to contradictory conclusions in the hands of different investigators.

He then applies Cauchy's index theorem. By applying this theorem in a familiar manner to the region bounded by the imaginary axis and a large circle over the left-hand half of the plane, he arrives at necessary and sufficient conditions for stability.

One of the conclusions deals with a mechanical system of one degree of freedom with positive mass a, positive spring constant c, and positive damping coefficient b, and with retarded (unit time lag) coefficient β. If the damping coefficient of such a system is greater than or equal to the absolute value of the retarded damping coefficient, the system is stable. This agrees with the results of Minorsky. H. Poritsky, U.S.A.

22. Karl Pflanz, "Distribution of wheel pressure in unsymmetric locomotive drives (Raddruckverteilung bei unsymmetrischen lokomotivantrieben)," *Schweiz. Bauztg.*, July 5, 1947, vol. 65, no. 27, pp. 367–370.

The moment which is produced by the forces of the coupling causes changes of the axle loading. It had been previously mentioned in the literature that these load changes are equally distributed over the two wheels of the same axle. Proof is given in this paper that this holds true not only for symmetrical locomotive drives such as nose suspended motors with pinion nose drives on each side, but also for unsymmetrical drives such as nose suspension with pinion drives on only one side.

Most of the paper is taken up by the derivation of the equations for the changes in axle loading for the following driving systems: (1) Nose suspension with pinion drive on one side and motor suspension not in center plane between wheels; (2) Disk drive (Brown-Boveri Scheibenantrieb); (3) B_o-B_o Locomotive (two trucks with two driven axles per truck). For the B_o-B_o Locomotive, Series 251 of the Loetschberg Railroad (Switzerland), the dynamic axle loading varies from 17.6 to 22.4 tons for a static axle loading of 20 tons. Robert Fehr, U.S.A.

23. A. I. Bellin, "Determination of the natural frequencies of the bending vibrations of beams," *J. appl. Mech.*, Mar. 1947, vol. 14, no. 1, pp. 1–6.

The natural frequencies of bending vibration are determined by a trial-and-error solution of a system of linear algebraic equations governing bending moments. Each trial solution is made for an assumed frequency. If the correct frequency has been assumed the final boundary condition will be satisfied.

The method is compared with a method due to N. O. Myklestad ["A new method of calculating natural modes of uncoupled bending vibration of airplane wings and other types of beams," *J. aero. Sci.*, Apr. 1944, vol. 11, pp. 153–163] in order to illustrate that the author's method requires less calculation effort. Applications to a wedge-shaped beam and to continuous beams are discussed. Stanley U. Benscoter, U.S.A.

24. Charles E. Crede and J. Paul Walsh, "The design of vibration-isolating bases for machinery," *J. appl. Mech.*, Mar. 1947, vol. 14, no. 1, pp. 7–14.

After deriving expressions for the several natural frequencies of vibration of a mass supported on four helical compression springs, the authors present their results on dimensionless co-ordinates and by means of an example show how to design a vibration-isolating base having three or more of its highest natural frequencies nearly equal and below a specified maximum. In addition to the spring characteristics, this method determines their vertical location and spacing relative to the center of gravity and radii of gyration of the supported mass. This more elaborate method is of particular value in isolating systems excited at low frequencies.

Of special interest are curves from which can be found the two rotational natural frequencies in a given principal vertical plane. Experimental results on a model are found to agree (within 10 per cent) with results from these curves. In other curves are presented lateral stiffness and lateral stability characteristics of helical compression springs with ends restrained so as to remain parallel. Alexander Yorgiadis, U.S.A.

25. Charles C. Kennedy and C. D. P. Pancu, "Use of vectors in vibration measurement and analysis," *J. aero. Sci.*, Nov. 1947, vol. 14, no. 11, pp. 603–630.

The paper deals with the analysis of experimentally recorded curves of amplitude and phase for points of a complicated structure such as an airplane, as functions of the frequency β of disturbing forces. Useful advice is given as to the arrangement of linear vibrators and pickups in such experiments. A main object is to separate the different normal modes of the structure and to outline their shape.

In order to facilitate the analysis, the authors use a type of complex vector representation of frequency responses similar to that commonly used in electrical engineering. If the top of the frequency response vector describes a curve with length of arc s, the derivative $ds/d\beta$ will have a maximum at points of resonance. A general method of identifying normal modes from the frequency vector plots consists of drawing the best circle possible through each approximately circular arc on which a maximum in $ds/d\beta$ is distinguishable. From the position and size of the circle the actual response and the damping in the corresponding normal mode may be determined. F. K. G. Odqvist, Sweden

26. N. W. McLachlan, "Vibrational problems in elliptical co-ordinates," *Quart. appl. Math.*, Oct. 1947, vol. 5, pp. 289–297.

This paper begins by considering the differential equation of vibration of a membrane of uniform tension and mass density, and with sinusoidal time displacement. By introducing elliptical co-ordinates, appropriate solutions are obtained for the cases: (1) A uniform, homogeneous, loss-free stretched membrane in the form of an elliptical ring; (2) water in a lake of uniform depth whose plan view is an elliptical ring.

Next the paper considers the differential equation of motion of a vibrating elastic plate of uniform thickness and mass density, and with sinusoidal time-displacement. By introducing elliptical co-ordinates appropriate solutions are obtained for the cases: (3) a uniform, homogeneous, loss-free, elastic elliptic plate; (4) a uniform, homogeneous, loss-free, elastic elliptic ring plate.

Finally two examples are given to show how specific approximate results can be obtained by using asymptotic formulas for some of the functions involved. The two examples involve an elliptical ring membrane and an elliptical plate.

N. O. Myklestad, U.S.A.

27. W. P. Targoff, "The associated matrices of bending and coupled bending-torsion vibrations," *J. aero. Sci.*, Oct. 1947, vol. 14, no. 10, pp. 579–582.

This paper presents a method for obtaining the bending and torsional vibrations of a beam by the use of matrices. The beam is divided into sections with concentrated masses connected by weightless elastic shafting. The shear forces, bending moments, bending slope angles, and bending deflections at each mass and section are expressed in matrix form. The bending matrix equation is obtained by successive matrix premultiplications of the successive dynamic elastic matrices.

By evaluating the matrix for a number of values of the frequency constant and plotting the imbalance of the desired end condition in terms of the frequency constant, the frequencies are determined as in Holzer's method for torsional systems. The coupled bending-torsion matrix is obtained by combining the associated torsion matrix of a beam section with the bending matrix. Methods for shortening the calculations are suggested and illustrated by means of a numerical matrix.

Fred C. Jonah, U.S.A.

Wave Motion, Impact, Seismology

(See also Revs. 18, 130, 131, 136, 178, 179, 187)

28. R. C. Binder and A. S. Hall, "An introduction to an analysis of gas vibrations in engine manifolds," *J. appl. Mech.*, Sept. 1947, vol. 14, no. 3, pp. 183–187.

The performance of an internal-combustion engine may be influenced favorably or unfavorably by the oscillations of the gas column in the intake and exhaust manifold, which may attain the amplitude of 3 to 4 psi. The treatment given these phenomena is based on the oscillations of a gas column in a simple tube, at one end of which is a "driving element" and at the other end a "terminal element." The concept of "impedance" is introduced, which is defined as the ratio of excess-pressure amplitude to the particle-velocity amplitude, or, in other words, the amount of excess pressure produced by unit particle velocity. The effect of damping, or attenuation factor, is explained.

The basic relations thus developed are applied to a simple engine induction system of a four-cycle internal-combustion engine, assuming an instantaneous opening of the inlet valve. The application of the method to more involved conditions is also indicated.

K. J. De Juhasz, U.S.A.

Acoustics

(See also Revs. 17, 70)

29. R. H. Nichols, Jr., "Flow-resistance characteristics of fibrous acoustical materials," *J. acoust. Soc. Amer.*, Sept. 1947, vol. 19, no. 5, pp. 866–871.

Aircraft cabins are often quieted by blankets of fibrous and very porous acoustical materials. The flow resistance (pressure drop across sample divided by linear air velocity) largely determines the amount of soundproofing attainable. This experimental study investigates the factors affecting the flow resistance.

It is found that for blankets of such porous materials as milkweed and kapok, and glass, cotton, and mineral wool, the flow resistance varies as $S^{(1+x)}/T^x r^2$, where S is the surface density, T the thickness, and r is the fiber radius. The exponent x varies from 1.0 for a random orientation of fibers to about 0.3 for the fibers lying parallel to the surface. Numerous illustrative data are given graphically

Vincent Salmon, U.S.A.

30. A. E. Bate and M. E. Pillow, "Mean free path of sound in an auditorium," *Proc. phys. Soc.*, July 1, 1947, vol. 59, part 4, no. 334, pp. 535–541.

The mean free path of sound in any rectangular enclosure, in a spherical enclosure, and in any cylindrical enclosure is shown to be $4V/S$ where V is the volume of the room and S its surface area. It is assumed that: (1) the sound field is completely diffuse; and (2) that the length of time T for which the averaging takes place is long enough to include a large number of reflections, thus making negligible the effect of the first and last reflections during the interval T.

The method takes into account wave packets emanating from all points in the enclosure, as distinguished from methods used by some previous investigators who arbitrarily specified location or directionality of the source, thus obtaining mean free paths different from $4V/S$.

Albert London, U.S.A.

31. L. J. Sivian, "High frequency absorption in air and other gases," *J. acoust. Soc. Amer.*, Sept. 1947, vol. 19, no. 5, pp. 914–916.

This is an account of measurements made in 1938 of the acoustic amplitude-absorption coefficient α between 15 and 500 kc, for several gases. The rms spread is about 25 per cent but the results, which are given as graphs, may be useful. No experimental details are given. Acoustic pressures were measured along an approximately plane progressive wave in a tube. Up to 40 kc these were supplemented by rate-of-decay measurements in plane standing waves.

The gases, tested at 76 cm and 26.5 C, were: dry O_2, N_2, $O_2 + N_2$, and $O_2 + N_2 + CO_2$; $O_2 + N_2 + CO_2 + H_2O$ (37 per cent relative humidity); $O_2 + N_2$ + varying CO_2. The mixtures, except the last, had normal air proportions. Differences in α between the first four were indistinguishable; the value of α followed the Stokes law, but about 50 per cent too high. Water vapor made α greatly exceed the Stokes value below 200 kc; above, the excess approached 50 per cent. At 89 kc, where CO_2 absorbs strongly, tests were made on $N_2 + O_2$ in normal air proportions plus 0 to 1.15 per cent CO_2. The increase in α due to CO_2, plotted against % CO_2, rose linearly to 1.2 db per ft. Thus CO_2 absorbs more than H_2O, in corresponding concentrations, at 89 kc.

A. O. Williams, Jr., U.S.A.

32. L. L. Beranek and H. W. Rudmose, "Airplane quieting: I—Measurement of sound levels in flight," *Trans. Amer. Soc. mech. Engrs.*, Feb. 1947, vol. 69, no. 2, pp. 89–95.

This is a discussion of presently available equipment for measuring and analyzing noise, and it reviews the types, characteristics, and uses of sound analyzers and noise meters. Useful information is given on such matters as the characteristics required for associated microphones and filters, the ability of many commercial noise meters to add anharmonic components, and the special techniques required for measuring the sound levels in airplanes.

Vincent Salmon, U.S.A.

33. L. L. Beranek, "Airplane quieting: II—Specification of acceptable noise levels," *Trans. Amer. Soc. mech. Engrs.*, Feb. 1947, vol. 69, no. 2, pp. 97–100.

The author calculates the ability of the voice to convey intelligence through noise, taking as the raw data the spectra of the speech and noise. The French-Steinberg concept of articulation index is used to obtain a plot of a standard speech spectrum in which the total area between the minimum and peak value curves roughly corresponds to the perfect transmission of intelligence.

The fraction of this area left after masking by the noise spectrum is taken as the measure of over-all articulation, 40 per cent being just acceptable. This procedure is valid when the two spectra are similar, and reasonable agreement with measured values is found. The interference of the noise may be measured by the arithmetic average of the decibel noise levels in each of the three octaves from 600 to 4800 cps. A table is given of these speech-interference levels as functions of distance and degree of speech loudness.

Vincent Salmon, U.S.A.

34. R. D. Fay, R. L. Brown, and O. V. Fortier, "Measurement of acoustic impedances of surfaces in water," *J. acoust. Soc. Amer.*, Sept. 1947, vol. 19, no. 5, pp. 850–856.

The authors study analytically and experimentally the extension to a water medium of the well-known tube method of measuring the acoustic impedance of surfaces. This extension requires that the boundary presented by the tube wall be so rigid that there will be a negligible energy transmission by the wall from one part of the sound field to another in the useful frequency range.

The authors show that the necessary conditions can be realized by proper choice of the material, diameter, and wall thickness of the tube, and report the results of phase, velocity, and pressure-amplitude measurements with two experimental impedance tubes: (1) a 6-ft length of 8-in-diam steel pipe with a $^1/_2$-in. wall; and (2) an 8-ft length of $5^1/_2$-in. OD aluminum tube with a $^1/_4$-in. wall. These tests confirm the theory. A number of experimental problems are discussed, including the need for high

stiffness in the exploring hydrophone structure and conductor lead arrangement, to avoid distortion of the sound field.

Ralph P. Glover, U.S.A.

35. Henry Primakoff and J. B. Keller, "Reflection and transmission of sound by thin curved shells," *J. acoust. Soc. Amer.*, Sept. 1947, vol. 19, no. 5, pp. 820–832.

The authors have derived in this paper a first approximation to the expression for determining the sound field due to a point source in the presence of a thin curved shell. Expressions for the reflection and transmission coefficients have then been calculated.

It is shown that the solution obtained reduces to the solutions obtained by using the Kirchhoff assumptions and the assumptions of geometrical acoustics, in special cases. It is considered that this agreement represents partial justification for the Kirchhoff theory.

The laws of reflection and refraction, the conditions for formation of point images and for change of phase at a focus are also derived.

R. G. Wilson, U.S.A.

Elasticity Theory

(See also Revs. 1, 5, 52, 53, 84, 96)

36. R. S. Rivlin, "Torsion of a rubber cylinder," *J. appl. Phys.*, May 1947, vol. 18, pp. 444–449.

An account is given of experiments designed to verify proposed forms of the stored-energy function for highly elastic materials such as rubber. A theory which is to be published elsewhere predicts that, in order to produce pure torsional deformation in a cylinder of such materials, not only must a torsional couple be exerted on the plane ends of the cylinder but also normal tractions must be exerted on these ends.

In the experiments reported it is assumed that the normal tractions can be measured by measuring the bulging of the rubber into small holes in a metal plate on one end of the cylinder. The experiments confirm a form of the stored-energy function equivalent to one proposed by M. Mooney [*J. appl. Phys.*, 1940, vol. 11, p. 582].

H. W. March, U.S.A.

37. L. Fox, "Mixed boundary conditions in the relaxational treatment of biharmonic problems (plane strain or stress)," *Proc. roy. Soc., London, Ser. A*, June 3, 1947, vol. 189, no. 1019, pp. 535–543.

The relaxation method is adapted in this paper to the solution of problems involving flexure or extension of flat elastic plates, where the boundary conditions are "mixed," i.e., displacements are specified at some points and tractions at others. This type of problem is usually difficult both for analytical treatment and for photoelastic experiment.

Two examples are given: the first, a square plate stretched by forces acting on two opposite ends, the forces being distributed according to a parabolic law; the second, a circular plate with a concentric hole, with uniform radial displacements imposed round the hole and no traction on the outer boundary.

C. T. Wang, U.S.A.

38. L. M. Milne-Thomson, "Stress in an infinite half-plane," *Proc. Camb. phil. Soc.*, Apr. 1947, vol. 43, part 2, pp. 287–288.

This note presents a general solution of the problems of two-dimensional elasticity for an isotropic body that covers the complex half plane, $R(z) > 0$. The solution is based upon the theorem that the components of stress and the components of displacement are derivable from two complex potential functions. Formulas for the potential functions are given for the case in which a body covering the entire z-plane is loaded by a single point force and a single finite point couple. Superposition then furnishes the general solution for the complete plane.

Perturbation functions are derived which must be added to the potential functions of the complete plane in order to obtain the potential functions of the half plane, with any loading throughout the interior and upon the free edge. The use of complex function theory renders the analysis remarkably concise.

H. L. Langhaar, U.S.A.

39. W. Prager and J. L. Synge, "Approximations in elasticity based on the concept of function space," *Quart. appl. Math.*, Oct. 1947, vol. 5, pp. 241–269.

The authors make use of the theory of linear function spaces to formulate the approximations to the boundary value problems of elasticity. The criterion of goodness of approximation is based on the strain-energy function:

$$W = {}^1/_2 C_{ijkl} E_{ij} E_{kl} \text{ (summed on } i, j, k, l = 1, 2, 3,) \ldots . [1]$$

where the constants C_{ijkl} satisfy the symmetry relations $C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}$ and the E_{ij} are the components of symmetric stress tensor, which are functions of rectangular Cartesian co-ordinates x_i. Thus, the elastic medium is anisotropic and the generalized Hooke's law has the form $e_{ij} = C_{ijkl} E_{kl}$.

The state of stress in the medium is given by a set of six functions E_{ij} determining a vector S in a metric manifold characterized by the positive definite form [1]. It turns out that the geometry of this manifold, in any linear subspace based on a finite number of vectors, is Euclidean.

The natural state S of an elastic body τ in equilibrium, in the absence of body forces, is determined by the following systems of equations, in which the commas preceding the subscripts denote differentiation with respect to the corresponding Cartesian variables:

(*a*) Equilibrium equations, $E_{ij,j} = 0$, in τ;

(*b*) Compatibility equations, $e_{ij,kl} + e_{kl,ij} - e_{ik,jl} - e_{jl,ik} = 0$, in τ;

(*c*) Boundary conditions, $E_{ij} n_j = T_i$ on the surface Σ of τ,

where n_j is the unit exterior normal to Σ, and T_i is the assigned stress on the part of the surface where the displacements u_i are not specified.

The authors consider approximate states $\bar{S}$ in which one or more of the conditions (*a*), (*b*), (*c*) are relaxed. The criterion for the goodness of approximation is that the "distance" $|\bar{S}\text{-}S|$ be small, the length S being defined by the inner product $S^2 = \int_\tau 2W d\tau = \int_\tau e_{ij} E_{ij} d\tau$. The theory is illustrated by obtaining approximate solutions for the torsion of a square prism.

The authors also compare their space with the function space introduced by S. Bergman, in which a point is determined by the components of displacement u_i. The principles of minimum potential energy and work are strengthened and are obtained without recourse to the calculus of variations. The connection of the authors' theory with E. Trefftz's method of solving two-dimensional problems of Dirichlet is also indicated.

I. S. Sokolnikoff, U.S.A.

40. C. B. Ling, "Torsion of a circular tube with longitudinal circular holes," *Quart. appl. Math.*, July 1947, vol. 5, no. 2, pp. 168–181.

The effects of drilling full-length longitudinal holes in a circular shaft under torsion are considered. The problem is solved for the case of holes of equal radii symmetrically spaced about the axis. The author improved on a somewhat similar previous solution by including the possibility of a central hole and deriving an equation for torsional stiffness.

In comparing different possible arrangements and sizes of

holes, the great accuracy necessary may be obtained by such a mathematical solution as this, which satisfies Laplace's equation with a harmonic function in the form of an infinite series. Examples are given of three different arrangements. For these the maximum stress occurs next to the outermost points of the eccentric holes. W. C. Johnson, Jr., U.S.A.

41. A. E. Green, "The flexure and torsion of aeolotropic beams," *Proc. Camb. phil. Soc.*, Jan. 1947, vol. 43, part 1, pp. 68–74.

The Saint Venant torsion and flexure problem for a uniform beam is extended to cover the case of a beam of anisotropic material having "orthotropic" symmetry. One axis of symmetry is along the length of the beam. The other two are not required to be parallel to the principal axes of the cross section, which may be of any shape.

The problem is reduced to that of finding seven canonical flexure functions (harmonic) which satisfy certain conditions on the boundary of a transformed cross section. For certain cross sections, such as polygonal or elliptical, the detailed solution may be carried out by methods already available.

Martin Greenspan, U.S.A.

42. C. Gurney and A. Hammond, "The swelling of an orthotropic circular tube," *J. appl. Mech.*, Sept. 1947, vol. 14, no. 3, pp. 188–190.

Many organic sheet materials swell, owing to moisture absorption, more in the direction of the thickness than in the plane of the sheet. In the present paper there is considered a tube made of such material, which is permitted to swell. Three boundary conditions are treated: stress-free peripheries, constant internal diameter, and constant external diameter. In all cases the stresses and displacements are determined readily from the equations of equilibrium and Hooke's law in terms of cylindrical co-ordinates. G. E. Hay, U.S.A.

43. Jane M. Dewey, "The elastic constants of materials loaded with non-rigid fillers," *J. appl. Phys.*, June 1947, vol. 18, pp. 578–581.

Formulas are derived for the average elastic constants of a heterogeneous body composed of an isotropic, elastic medium having embedded in it a large number of elastic spheres. The development is based on J. N. Goodier's solution of the elasticity equations for the strain in an isotropic body containing a single elastic sphere. The average constants are expressed in terms of the sphere radius, the fraction of the total volume occupied by the spheres, and four elastic constants (two for the spheres and two for the surrounding medium). R. D. Mindlin, U.S.A.

Experimental Stress Analysis

(See also Revs. 60, 74, 83, 88, 121)

44. G. Sachs, C. S. Smith, J. D. Lubahn, G. E. Davis, and L. J. Ebert, "Nondestructive measurement of residual and enforced stresses by means of x-ray diffraction," *Welding Res. Supplement*, Jan. 1947, vol. 12, no. 1, pp. 26–49.

Measurements employing x-ray diffraction techniques were made to determine the principal stresses in flat 60-deg V-notched tensile bars of SAE 4130 steel and 24S-T aluminum alloy, within and beyond the elastic range, and to survey residual stresses in a butt-welded joint in thin SAE 4130 tubing. Details and difficulties in the experimental techniques, apparatus, and methods of interpretation are given. It was necessary to condition the materials by optimum heat-treatments before suitable diffraction patterns could be obtained.

In the elastic range, measurements of surface stresses in notched bars were in agreement with theoretical and photoelastic results, and the ratio of transverse to longitudinal stress was independent of load but varied with depth of notch. Longitudinal stresses in the plastic range at several stress levels confirmed previous theories.

The authors infer that stresses on the surface do not increase as rapidly as those in the interior when the average stress is in the plastic range. However, two factors make some of the numerical values of peak stress doubtful: (*a*) Measurement by x-ray diffraction of the peak stress in a region of high stress gradient proved impractical; (*b*) in the plastic range it was found necessary to reduce the load 15 per cent below the maximum to eliminate creep during the x-ray exposure, necessitating an arbitrary calculation to correct for the stress existing at the maximum load.

For the welded tubing the x-ray analyses did not prove suitable to determine residual stresses in the heat-affected parent metal, and erratic variations of stresses were observed in the deposited weld metal, indicating that the method has restricted applicability for determining residual stress in alloy steel weldments. Thomas J. Dolan, U.S.A.

45. W. H. Pickering, "Reluctance gages for telemetering strain data," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 4, no. 2, pp. 74–78.

The author discusses three types of strain gages with reference to their use in telemetering systems, particularly systems which contain radio links, and concludes that: (1) Wire resistance gages have many advantages but have the disadvantage of a very small electric output and so require amplification before the signal is large enough to modulate the telemetering transmitter. (2) Radio-frequency reactance gages (capacity pickups or air-cored inductances) are simple to build and can modulate a radio-frequency carrier directly, but require very good oscillator stability and are difficult to multiplex. (3) Audio-frequency reactance gages (reluctance gages) which vary the air gap in the iron core of a coil are the most satisfactory for telemetering. They are a little more complex to build but there are no modulation difficulties and multiplexing is very straightforward. B. F. Langer, U.S.A.

46. Greer Ellis, "Stress determination by brittle coatings," *Mech. Engng.*, July 1947, vol. 69, no. 7, pp. 567–571.

This is primarily an introduction to the use of brittle coatings in stress analysis for those to whom the use of this very practical tool is comparatively new. However, several points are made which should be of interest to workers already using brittle coatings.

For example a figure of 0.0006 unit strain is given as the degree of sensitization resulting from the use of dye etchant under load, which, when superimposed on the normal strain sensitivity, permits the measurement of extremely low strains. The use of brittle coatings outdoors, which can be accomplished successfully if the test schedule is planned with regard to normal daily temperature fluctuations, is discussed. A method is proposed for estimating the magnitude of residual stresses in a rough quantitative way, based on the extent of the (fully developed) pattern relative to the diameter of a hole drilled into the residually stressed surface.

Charles W. Gadd, U.S.A.

47. H. Schaevitz, "The linear variable differential transformer," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 4, no. 2, pp. 79–88.

This paper presents test data and discussion covering the use of the subject device as an electrical displacement pickup. Some particular advantages shown are high output (comparable with practical slidewire installations), and mechanical simplicity and ruggedness, which allows its use in some applications where resist-

ance-type pickups would be impractical. The most serious limitation shown is the inherent frequency sensitivity of the device, which requires careful selection of input or "carrier" frequency to provide sufficient flat sideband width for accurate dynamic displacement measurements.

The test data presented cover the use of several sizes of pickups for measuring full-scale displacements of from 0.0015 in. to 0.5 in. with linear outputs ranging approximately from 1 to 15 volts per in. depending on input voltage, carrier frequency, and load impedance. Henry W. Foster, U.S.A.

48. P. B. Walker, "Stability of an aircraft structure in a strength test frame," *J. roy. aero. Soc.*, Aug. 1947, vol. 51, no. 440, pp. 704–714.

The purpose of this paper is to acquaint test engineers with the need for careful analysis of the loading devices and methods of support in a given test, so that unstable systems of applied loads and reactions cannot develop during test of a complete airplane or its components. Particular problems are discussed in a rather vague manner. Difficulties to be expected are suggested and solutions are implied but not presented explicitly.

Joseph S. Newell, U.S.A.

49. E. T. P. Neubauer and O. W. Boston, "Torsional stress analysis of twist-drill sections by membrane analogy," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, no. 8, pp. 897–902.

The construction and operation of the apparatus and the experimental procedure are detailed. Results are given in tables and membrane contour maps. Twenty-one drill designs are considered and relative design efficiencies are indicated. The authors do not discuss the justification of applying the membrane analogy in the solution of this problem, which involves nonprismatical bars. George H. Lee, U.S.A.

50. A. W. Brunot and W. B. Goddard, "Static-load tests on an aircraft gas turbine to simulate loads produced by rapid plane maneuvers," *J. appl. Mech.*, Mar. 1947, vol. 14, no. 1, pp. 15–20.

Static structural tests of an aircraft gas turbine are described. The design loads due to acceleration and gyroscopic action were simulated individually and in combination by applying a number of concentrated loads, so placed and proportioned as to approximate the design values of shear and moment in the test structure.

Henry W. Foster, U.S.A.

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 12, 20, 23, 24, 40, 41, 75, 77, 83)

51. T. K. Wang, "Elastic and plastic bending of beams," *J. aero. Sci.*, July 1947, vol. 14, pp. 422–432.

An analysis of bending stresses is developed for material exhibiting nonlinear stress-strain relations. Aircraft sections are treated. Remote fiber stresses for pure bending are determined as a function of the moment divided by a quantity which is similar to the section modulus. An empirical equation for the stress-strain relationships is suggested. Curves for various shaped beam sections are given. Clayton O. Dohrenwend, U.S.A.

52. Hans Ziegler, "An extension of the bending theory (Eine erweiterung der technischen biegungslehre)," *Schweiz. Bauztg.*, Jan. 11, 1947, vol. 65, no. 2, pp. 17–20, and Jan. 18, 1947, vol. 65, no. 3, pp. 30–32.

Straight homogeneous beams are considered. The cross section is a narrow rectangle with a constant shorter side. Small variations are permitted in the length of the longer side. The loads act on the shorter side, so the state of stress is approximately plane. Expressions are sought for the stress components, in the form of a power series in z (the co-ordinate varying along the longer axis of the cross section) with coefficients which are functions of x (the co-ordinate varying along the axis of the beam). In an earlier paper the first coefficients and the second coefficients were determined approximately by a consideration of the orders of magnitude of the terms in the equations of equilibrium and compatibility. In the present paper the third coefficients are determined similarly. G. E. Hay, U.S.A.

53. Henry L. Langhaar and Clarence R. Smith, "Stresses in cylindrical semimonocoque open beams," *J. aero. Sci.*, Apr. 1947, vol. 14, pp. 211–220.

Expressions are developed for the normal and shearing stresses in the skin and two stringers of an open circular cylindrical beam, by minimizing the strain energy. Conventional assumptions of equilibrium, continuity, and elastic action are used, and hoop stresses in the skin and shearing stresses in the longerons are assumed negligible. An adaptation of Lagrange's method of undetermined multipliers is employed in the minimizing process.

Several curves compare theoretical values with results obtained from tests on a 248-T Alclad cylinder 168 in. long, 30 in. in diameter, and 0.027 in. thick, supported as a cantilever and loaded with either a force or a torque at the free end. Strains were measured by electric resistance gages for conditions of no cutout, and one cutout ranging from 30 deg by 14 in. to 90 deg by 29 in., reinforced in various ways. There was reasonable agreement between tests and theory. The authors conclude that the elementary flexure formula applies in the cutout interval for symmetrical bending, and that there is a fairly rapid transition to the undistorted stress pattern in the closed cylinder.

Glenn Murphy, U.S.A.

54. D. H. Cornell and J. R. Beatty, "Laboratory testing of rubber torsion springs," *Trans. Amer. Soc. mech. Engrs.*, Oct. 1947, vol. 69, no. 7, pp. 799–804.

This is a report of tests on cylindrical rubber torsion springs consisting of flexible rubber, bonded between two concentric cylindrical metal shells, with the unit loaded in torsion as a spring or mounting. The tests results include: static torque-windup (angle of twist) and static modulus-torque curves; dynamic modulus-torque curves for frequencies of 1 to 3 cycles per second; hysteresis; fatigue life at stresses up to 360 psi in shear; and accelerated creep-time curves.

Elastomers from 35 to 60 durometer hardness were used, and some results are presented for temperatures from −30 F to 150 F. A method is also presented for computing static and dynamic moduli of any rubber torsion spring from the dimensions and fundamental data for the rubber. Henry A. Lepper, Jr., U.S.A.

55. Alexander Hrennikoff, "Theory of inelastic bending with reference to limit design," *Proc. Amer. Soc. civ. Engrs.*, Mar. 1947, vol. 73, pp. 255–289.

This paper concerns the analysis of beams loaded beyond the elastic limit. The fundamental relations between the beam stresses and strains during such loading are presented in a clear and concise manner. The major assumption used is that the beam cross sections remain plane throughout the entire range of flexure.

The integrals which are needed for the analysis of beams loaded beyond the elastic limit are developed for mild-steel structural I beams, and a brief table of the numerical values of these integrals is included, as well as tables of similar integrals for a greater range of parameters. Several numerical examples are given.

The author presents a relatively detailed comparison of his results with the corresponding results obtained by "limit design" as presented by Van den Broek ["Theory of limit design," *Trans.*

Amer. Soc. civ. Engrs., 1940, vol. 105, p. 638]. In general, it is shown that for a mild steel which has a strain-hardening region the results of limit design err somewhat on the safe side, although some cases are suggested for which limit design may prove unsafe. A. W. Rankin, U.S.A.

56. Harry A. Williams, "Pure bending in the plastic range," *J. aero. Sci.*, Aug. 1947, vol. 14, pp. 457–470.

This is a graphical solution of the problem, similar to the analytic solution by William R. Osgood ["Plastic bending—further considerations," *J. aero. Sci.*, July 1945, vol. 12, pp. 253–262, 272]. The author plots two curves derived from the stress-strain curve: f against $\int fde$, and f_m against $\int_0^{e_m} fde/e_m^2$, where f is the stress, e the strain, and the subscript m denotes maximum values, at the extreme fiber. Most of the paper and the applications are confined to symmetrical sections and stress-strain curves that are identical in tension and compression. Approximate solutions are discussed.

The author considers "that the ultimate resisting moments of beams are approximately proportional to the ultimate tensile strengths of the materials involved, if the stress-strain curves are of the same general form." A comparison is made between theoretically predicted strengths and actual (tested) strengths of aluminum alloys 195-T6 and 356-T6, and magnesium alloy H-HT. William R. Osgood, U.S.A.

57. Enrique Butty, "The simply supported beam of rectangular section and great height (Sobre la viga simplemente apoyada con sección rectangular de gran altura)," *Ciencia Tecn.*, Oct. 1947, vol. 109, no. 544, pp. 195–227.

The author argues that Ritz's method, as used by Timoshenko for rectangular plates, is better suited to the solution of beams of great height than the finite difference method used by H. Bay, and that good accuracy can be obtained much faster. The author works out general formulas for both methods and applies them in great detail to the case of a beam loaded on the lower boundary. A. J. Durelli, U.S.A.

58. H. F. Ross, "Application of tables for helical compression and extension spring design," *Trans. Amer. Soc. mech. Engrs.*, Oct. 1947, vol. 69, no. 7, pp. 725–734.

This is a tabular compilation, for design use, of data on cylindrical helical springs of circular cross section, with examples and a discussion of various factors modifying the data. Spring constant (lb per in.) per turn and maximum load are shown, for each spring size. The maximum load is based on a stress of 100,000 psi, and includes correction of the simple formula to allow for shear load and curvature. Design of compression and tension springs of this type is discussed in relation to average service conditions, maximum design load eliminating the curvature correction, and design for fatigue. W. C. Johnson, Jr., U.S.A.

Plates, Disks, Shells, Membranes

(See also Revs. 12, 64, 65, 66, 71, 81)

59. Eric Reissner, "On bending of elastic plates," *Quart. appl. Math.*, Apr. 1947, vol. 5, pp. 55–68.

An account in simpler and more general form is given of the author's earlier work ["The effect of transverse shear deformation on the bending of elastic plates," *J. appl. Mech.*, June 1945, vol. 12, no. 2, pp. 69–77]. A theory for bending of thin elastic plates is presented, which takes into account the effects of transverse shear deformability for plates of homogeneous or nonhomogeneous construction. The elastic properties in the direction perpendicular to the plane of the plate may or may not be different from those in directions parallel to the plane of the plate.

The author develops differential equations in both rectangular and polar co-ordinates, which relate the bending couples, twisting couple, and transverse shear-stress resultants to the surface load per unit area and the lateral deflection. An analysis is presented for the bending of a cantilever plate by a terminal transverse load, with a comparison of exact and approximate values for an isotropic plate. It is shown that, for very thin plates, the horizontal shear may be larger than the transverse shear even for isotropic plates. Samuel Levy, U.S.A.

60. Walter C. Voss, Dean Peabody, Jr., Howard R. Staley, and Albert G. H. Dietz, "Thin-shelled domes loaded eccentrically," *Proc. Amer. Soc. civ. Engrs.*, Oct. 1947, vol. 73 pp. 1173–1195.

This paper gives a description of tests made on a shallow thin-shelled dome of plaster of Paris subjected to concentrated eccentrically applied gravity loads, as well as sand loads distributed uniformly over an eighth segment and a quarter segment. Strains were measured on the upper surface at points near to and far from the applied loads, and were converted to stresses by the usual elastic stress-strain relationships.

The tests are compared with a mathematical analysis made by Eric Reissner ["Stresses and small displacements of shallow spherical shells," *J. Math. and Phys.*, Feb. 1946, vol. 25, no. 1, pp. 80–85] of the stresses caused by a concentrated load applied at the crown of a shallow thin-shelled spherical dome. The tests showed that stresses of any magnitude are essentially local ones, and that this analysis applies approximately to an eccentric load, provided it is applied at some distance from the foundation or from any hole in the shallow dome. I. A. Wojtaszak, U.S.A.

61. H. M. Westergaard, "New formulas for stresses in concrete pavements of airfields," *Proc. Amer. Soc. civ. Engrs.*, May 1947, vol. 73, pp. 687–701.

The concrete pavement of an airfield is regarded in this analysis as a slab supported on an elastic subgrade, the reaction of which is proportional at every point to the deflection of the slab at that point. The load is a pressure uniformly distributed over an oblong "footprint" area of a tire. In one part of the analysis the outline of this contact area is assumed to be an ellipse, but general solutions are given for any other symmetrically shaped contact area.

Three positions of this load are considered, (1) when the load is in the interior of a slab panel at a considerable distance from the boundaries of the panel, (2) when the load is at a free edge of a panel, and (3) when the load is at the boundary of a joint in the slab which permits partial transfer of the load to an adjacent panel.

In each of these cases formulas are given for the calculation of the maximum bending stress at the bottom of the slab under the load, and for the deflection of the slab within or near the loaded area. The formulas are derived from basic solutions published previously by the author, by means of Maxwell's theorem of reciprocal deflections and an extension of it to include reciprocity of curvatures, that is bending stresses in the slab, which holds true in this particular case. Considering the complexity of the problem the formulas lend themselves well for practical calculations and their use is illustrated in the paper by numerical examples. M. Hetényi, U.S.A.

62. W. B. Stiles, "Bending of clamped plates," *J. appl. Mech.*, Mar. 1947, vol. 14, no. 1, pp. 55–62.

A method is described for obtaining an approximate solution to the problem of a flat plate subjected to lateral load, when either all or part of the boundary is clamped and the remainder is simply supported. The solution is based on Trefftz's approximation

[*Proc. Second Int. Cong. for Applied Mech., Zurich* (in German), 1926, pp. 131–137] in which the differential equation is exactly satisfied but the boundary requirement of zero slope at the clamped edges is relaxed, and follows Weinstein [*Mémorial des Science Mathématique*, 1937, vol. 88, pp. 1–62] in using for correction functions the normal modes of freely vibrating membranes and the deflection surface of membranes subjected to static loading.

The author discusses rectangular plates with uniform and with central point loads, clamped on all four edges for the case of a rectangular plate, and clamped on two adjacent edges with simple-support on the remaining two edges for the case of a square plate. Experimental results on a $14 \times 28 \times {}^{1}/_{8}$-in. "clamped" rectangular steel plate with a central point load were between the calculated results for a simply-supported plate and those for a clamped plate, approximating the latter much more closely than the former. Samuel Levy, U.S.A.

63. Alfred Pfeiffer, "A note on the theory of corrugated diaphragms for pressure-measuring instruments," *Rev. sci. Instrum.*, Sept. 1947, vol. 18, no. 9, pp. 660–664.

Dimensional analysis is applied to a series of pressure tests of corrugated diaphragms, in which a correlation is sought between the diameter, thickness, modulus of elasticity, applied pressure, and deflection. A fairing curve is thus obtained, approximating test data. The influence of the material and of corrugation shape are not investigated. Leon Beskin, U.S.A.

Buckling Problems

(See also Rev. 76)

64. C. F. Kollbrunner, "Buckling of uniformly compressed rectangular plates in the elastic and plastic ranges (Die Ausbeulung von durch einseitingen, gleichmässig Verteilten Druck beanspruchten Blechen im elastischen und plastischen Bereich)," *Schweiz. Bauztg.*, Feb. 22, 1947, vol. 65, no. 8, pp. 95–97.

The main purpose of this article is to call attention to the series of stability tests conducted at the Swiss Federal Institute of Technology. A brief review is presented and references are given to work completed on concentrically and eccentrically loaded bars of rectangular section, on the free legs of angles subjected to compression, and on the elastic and plastic buckling of uniformly compressed rectangular plates. A program is outlined for further tests on plates under various loading conditions and on plate girders with stiffeners, intended to establish simple and generally useful instructions for the calculation of plate girders.

Formulas, with comments as to application, are presented for buckling in the elastic and plastic ranges of rectangular plates simply supported along two opposite sides perpendicular to the direction of compression, and having various boundary conditions along the other two sides. Agreement is noted between these more general formulas and the solutions for elastic buckling of Bleich and Chwalla. For buckling in the plastic range, an experienced calculator can obtain satisfactory accuracy by three successive approximations. J. H. Meier, U.S.A.

65. C. C. Wan, "Face buckling and core strength requirements in sandwich construction," *J. aero. Sci.*, Sept. 1947, vol. 14, pp. 531–539.

This analysis of a bonded sandwich panel is based upon the assumption that there are initial sinusoidal waves in the skin, with amplitude proportional to the square of the wave length and inversely proportional to the skin thickness. Equilibrium states for cases of uniaxial and biaxial compression are then obtained by application of the principle of virtual work. The ultimate load for a panel is considered to be that for which the ultimate tensile stress of the core is developed in the direction perpendicular to the skin, the wave length of initial wrinkles being taken as the value that furnishes maximum transverse tensile stress.

Derived charts are presented for panels with 24S-T81 and 75S-T skins and balsa cores. Some test data are given as a guide for the selection of an "initial waviness factor" that occurs in the theory. Henry L. Langhaar, U.S.A.

66. R. G. Sturm, "Stability of thin cylindrical shells in torsion," *Proc. Amer. Soc. civ. Engrs.*, Apr. 1947, vol. 73, pp. 471–495.

This analysis starts with the general differential equations for thin-walled cylinders, which are simplified by neglecting certain terms and are solved by assuming that the radial displacement is a particular function of the co-ordinates. This function is chosen to satisfy the end conditions, which are designated as "simply supported," and to give a buckled shape which conforms to that obtained in tests. It should be noted that the author's definition of "simply supported" differs somewhat from that generally used.

The critical buckling stress in shear, τ_c, is expressed by the formula $\tau_c/E = K_D(t/D)^2$ where K_D depends upon both the length-diameter ratio and the thickness-diameter ratio. A fairly complete family of curves covering both short and long tubes is given, from which K_D may be determined.

Approximate procedures are given for estimating the effect of initial out-of-roundness, for extending the results to cylinders with fixed ends, to cylinders with circumferential and longitudinal stiffeners, and to cylinders in bending with transverse shear. A comparison between these theoretical results and experiments reported in the literature indicates that the agreement is satisfactory when allowances for out-of-roundness are considered. Dana Young, U.S.A.

67. T. K. Wang, "Torsion and shear effects of members upon general instability of semimonocoque structures under compression," *J. appl. Mech.*, Sept. 1947, vol. 14, no. 3, pp. 177–182.

The author computes the buckling load for general instability under end compression of a circular cylinder reinforced by equally spaced stringers and rings. It is assumed that Hooke's law holds, that the rings buckle in a plane normal to the axis of the cylinder, that stringers buckle both radially and tangentially, and that neither local nor panel instability occurs.

A general solution is obtained by assuming a pattern of rectangular buckles for the radial and tangential displacements, which are described by a Fourier series and substituted in an energy equation in accordance with the Rayleigh-Ritz-Timoshenko method. The energy equation includes terms for the torsional and flexural energy in rings and stringers and for the shear energy in the sheet.

The solution is simplified for buckles with wave lengths that do not exceed the stringer and ring spacing, and curves of buckling load are given as a function of the dimensional and elastic constants of the structure for the case in which the rings buckle into a ellipse. Walter Ramberg, U.S.A.

68. Bruno A. Boley, "Numerical methods for the calculation of elastic instability," *J. aero. Sci.*, June 1947, vol. 14, pp. 337–350.

This is a contribution to the theory of buckling. It is of more interest and possible use to research than to practical applications, and is of special interest to those concerned with buckling of sheet-stringer combinations.

A general review is made of Southwell's relaxation method. A theoretical background is developed for the use of operations tables in the application of the methods outlined in the paper.

This involves a summary of the theorems relating operations tables to conditions of structural stability, neutral equilibrium, and structural instability. Three methods—the determinant method, the convergence method, and the energy method—for computing buckling loads are then outlined.

These methods are demonstrated in a synopsis of a numerical computation for a test specimen consisting of sheet, stringer, and ring combination. The computed buckling load was about 5.5 per cent higher than the buckling load determined by actual tests on the specimen. R. G. Sturm, U.S.A.

69. William Prager, "The general variational principle of the theory of structural stability," *Quart. appl. Math.*, Jan. 1947, vol. 4, pp. 378–384.

This paper is concerned with the general problem of structural stability in the elastic or plastic range. The point of view of given initial stresses is taken. This enables the treatment of thermal stresses and plastic problems together with elastic problems, in a unified theory.

The general problem of structural stability is reduced to an eigen-value problem for the displacements from a configuration of indifferent equilibrium to a neighboring configuration of this type. A variational principle is then derived which is equivalent to the eigen-value problem thus formulated. As an example of the application of this principle the lateral buckling of an unevenly heated lamina is treated. C. C. Lin, U.S.A.

70. L. T. Wyly, "Rational design of sections for short compression members of steel," *Bull. Amer. rly. engng. Assoc.*, June–July 1947, vol. 49, no. 467, pp. 67–86.

A review is given of some theoretical and experimental data regarding the buckling strength of plates and angles, and the behavior of lacing bars. From this evidence specifications are proposed for the design of short, built-up, steel columns to prevent premature local failure. Dana Young, U.S.A.

71. S. Pines and G. Gerard, "Instability analysis and design of an efficiently tapered plate under compressive loading," *J. aero. Sci.*, Oct. 1947, vol. 14, no. 10, pp. 594–600.

A rectangular isotropic elastic plate with simply supported edges $x = 0, a$ and $y = \pm b/2$, has no normal loading on the edges $y = \pm b/2$, while the compression in the x direction N_x is a decreasing exponential function of x. The plate is tapered so that the cube of the thickness is proportional to N_x, it being assumed on intuitive grounds that this gives the most efficient form for resistance to buckling. Using the energy method and assuming a sinusoidal deflection, the author determines the value of N_x for a wide range of aspect ratios and taper factors. An empirical method is proposed for applying the theory in cases in which the elastic limit is exceeded. H. L. Langhaar, U.S.A.

72. F. R. Shanley, "Inelastic column theory," *J. aero. Sci.*, May 1947, vol. 14, pp. 261–268.

In 1889–1895 Considère and Engesser suggested that the instability of short columns could be calculated by substituting the tangent modulus of elasticity for Young's modulus. Criticism of this led Engesser to modify it in favor of what has become known as the double-modulus theory. These suggestions were revived by von Kármán in 1910, when he substantiated the double-modulus theory by carefully controlled experiments and gave explicit formulas for some sections.

The author shows that the von Kármán analysis of the ideal pin-ended column under stresses above the elastic limit calls for further generalization. Within a certain load range immediately prior to reaching true instability at the double-modulus load, there can exist stable configurations in which the column bends under axial load alone. In the short column range column instability is reached, not abruptly as in the elastic range, but progressively through a certain transition range of loads.

In fact, there is an infinite family of load-transverse deflection curves starting from each point in this load range, which extends from the tangent-modulus load to the higher double-modulus load. The upper envelope is the load-deflection curve starting from the double-modulus load, and all curves of the family are asymptotic to this at large deflections. These intermediate curves differ from the upper envelope in that they require an axial load increasing with deflection.

The author proposes on the basis of a simplified column model that column collapse may occur by large deflections in this intermediate range below the double-modulus load, and thus he suggests that the tangent-modulus load, as a lower limit, be used in design in place of the double-modulus load. An illuminating discussion by von Kármán, which presents the general implications of the author's contribution, appears at the end of the article. J. M. Frankland, U.S.A.

Joints and Joining Methods

(See also Rev. 116)

73. R. L. Fefferman and H. L. Langhaar, "Investigations of 24S-T riveted tension joints," *J. aero. Sci.*, Mar. 1947, vol. 14, pp. 133–147.

This paper presents data from tension tests on over 200 specimens, with over 100 combinations of sheet thickness, rivet sizes, and patterns in lap and butt joints. Most of the specimens are lap joints. Some have staggered, some tandem, rivet patterns.

By assuming that rivets of unequal diameter in a joint carry equal bearing stresses rather than equal shearing stresses as the ultimate load is approached, and by using an empirical stress concentration factor, the authors show a good correlation between test data and curves derived from the elementary theory for tension joints. Curves are included for evaluating the factors, specific suggestions are made for proportions to be used to obtain high joint efficiencies, and simple procedures to be followed in the design of tension joints are outlined. J. S. Newell, U.S.A.

74. Arnold W. Hendry, "The testing of structural connections," *Engineering*, Sept. 12, 1947, vol. 164, no. 4259, pp. 261–263.

An experimental investigation to furnish information on the stability and stress distribution in various rigid-knee joints is described. Strain and deflection measurements, photoelastic analysis, destructive load tests, and analytical thought have been combined to arrive at design criteria suitable for portal-frame bents. Details concerning the experimental procedure and information on the instruments used is not included, but the paper is well illustrated and gives essential data covering the specimens and results. Herman Tachau, U.S.A.

75. A. R. C. Markl, "Fatigue tests of welding elbows and comparable double-mitre bends," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, no. 8, pp. 869–879.

The article summarizes accepted knowledge and practice in stressing piping subjected to flexing. Data are given for full-scale fatigue tests, in bending transverse to the plane of curvature, of welding elbows and double-mitre bends. Comparisons are made with Hovgaard's stress-intensification factor, as well as other theories. Fatigue curves and photographs of failures are shown.

Empirical constants for a simple relationship between endurance, strength, and number of cycles are given for elbows and mitre bends, for bending in and transverse to the plane of curvature. Welding elbows have a fatigue life for full stress reversal

in bending in this plane 2.4 times that sustained by the mitre bend, and 13 times the mitre bend for transverse bending. Internal pressures in the piping did not affect the results.

E. A. Brittenham, Jr., U.S.A.

Structures

(See also Revs. 48, 53, 67, 74, 116, 119)

76. W. W. Troxell and H. C. Engel, "Column characteristics of sandwich panels having honeycomb cores," *J. aero. Sci.*, July 1947, vol. 14, pp. 413–421.

The column characteristics of panels having metal faces and honeycomb cores are presented in the paper. In predicting the buckling loads, the authors use the Engesser formula, which takes into account both the bending and shearing deformations of the column. The formula is valid for columns that are weak in shear, and yields good approximate results for most sandwich panels where the thickness of the faces is small as compared to the thickness of a core which is weak in shear. When the compressive stress in the faces is beyond the elastic limit, a reduced modulus of elasticity is used, which is obtained by assuming the straight line short column curve.

Tests, carried out for a series of sandwich panels having 24S-T and 75S-T aluminum-alloy faces and honeycombs made of cotton fabric impregnated with phenolic laminating varnish to a resin content of 50 per cent, are in reasonable agreement with the computed values. Optimum panel construction, i.e., the ratio of face thickness to total thickness which yields the maximum strength-weight ratio, is also discussed. C. T. Wang, U.S.A.

77. J. F. A. Broeckhoven, "Contribution to the calculation of arch bridges in reinforced concrete with rigid arch and deck (Bijdrage tot het berekenen van boogbruggen van gewapend beton met stijve boog en stijf rijdek)," *Ingenieur, 's Grav.*, Oct. 3, 1947, vol. 59, no. 40, pp. 73–79.

An approximate calculation of the above-mentioned type of bridge is presented under the assumptions: (1) that the number of suspension bars is infinite; (2) that the elongation of the suspension bars, of the webbing, and of the arch itself is zero. The assumptions are discussed, corrections are given, and the errors are estimated. R. G. Boiten, Netherlands

78. Charles Doveton Crosthwaite, "The corrected theory of the stiffened suspension bridge," *J. Instn. civ. Engrs.*, Feb. 1947, vol. 27, no. 4, pp. 470–496.

A modification of the "deflection" theory is given which takes into account the horizontal as well as the vertical displacements of the cable, assuming that the cable is parabolic in shape. A trigonometric series expansion of deflections and moments is used, with the coefficients determined by Southwell's relaxation procedure. Nonuniform stiffening trusses, tower deflections, hanger extensions, and lateral wind forces may all be considered.

As an illustration, the process is applied to the Manhattan Bridge. For this case, however, the difference in moments from the orthodox "deflection" theory proves to be negligible.

N. M. Newmark, U.S.A.

79. S. Levy, "Computation of influence coefficients for aircraft structures with discontinuities and sweepback," *J. aero. Sci.*, Oct. 1947, vol. 14, no. 10, pp. 547–560.

This paper presents a theoretical and apparently practical procedure for computing the normal modes of vibration of an airplane part, such as a wing having discontinuities, cutouts, or sweepback, by the use of influence coefficients, calculated by Castigliano's theorem. The author points out that where the approximations as to stress distribution may be in error by ± 10 per cent, the strain energy and therefore the deflection or the influence coefficient should be in error by no more than 1 per cent. Hence he feels free to introduce as approximations in stress analysis the usual simplifying assumptions as to separate action of connected parts.

The other simplifying procedure introduced is the idea of "structural units." The deflection at any desired point is computed as the contribution of several structural units between that point and the fuselage. Structural units have boundary lines along sections where discontinuities do not exist and stresses are determinate. Discontinuities are isolated well within these structural parts so that the elastic energy may be approximated most readily.

In order to compute normal modes of vibration by a system of iteration it is necessary to replace the mass of the structural part by a system of concentrated masses. The author shows that these masses must produce the same deflection or influence coefficient as the distributed weight of the structural part. To achieve this effect he demonstrates that the total mass and the center of gravity of the substitute masses must be the same as that of the distributed mass. Also the moments of inertia and product of inertia of the substitute masses must replace these values of the distributed mass. Based upon this substitution the author shows in an appendix how normal modes of vibration may be computed.

L. E. Grinter, U.S.A.

80. George Winter and Minglung Pei, "Hipped plate construction," *J. Amer. concr. Inst.*, Jan. 1947, vol. 18, pp. 505–531.

The paper discusses and illustrates a method of design and construction of structures, adaptable to bunkers, long span roofs, hangars, etc. The structures consist of reinforced concrete slabs joined rigidly at various angles without the aid of beams or girders. The inclination of the slabs is such that the main loads are carried in the plane of the slabs, which, being quite deep, are capable of bridging considerable spans. The elimination of beams and girders leaves smooth surfaces which makes for simple formwork as well as a probable saving in material.

The unusual feature in the design of these slabs is due to the shearing forces set up along the intersection of two adjacent slabs. These shearing forces are due to bending about the strong axes of the slabs and they in turn cause normal forces on a section through the slab. The equations for these forces are derived in a form identical to the three-moment equation. It is also shown how they can be determined by the moment distribution method.

The method of design is illustrated by carrying through the main design features of a saw-toothed-type roof structure covering an unobstructed space of 72 × 150 ft.

Nicholas Di Pinto, U.S.A.

81. M. Davin, "Application of the statics of membranes to the study of dams (Application de la statique des voiles minces a l'etude des barrages)," *Ann. Ponts Chauss.*, Jan.–Feb. 1947, vol. 117, no. 1, pp. 47–83.

This paper develops a method for calculating the stress distribution in a dam having the form of a thin shell whose middle surface is an ellipsoid of revolution. An "extensional" solution is adopted, stresses arising from bending or twisting being neglected. Under these conditions the stresses can be calculated from equilibrium alone without reference to "elasticity."

The calculations are developed in two stages: (1) A particular solution is obtained which balances the applied forces, but does not satisfy the boundary conditions at the free edge; (2) an "internal" distribution is found which enables the edge conditions of zero stress to be satisfied. The first solution is obtained from that for a sphere by a transformation of points and forces for

which equilibrium relations are invariant. For the second solution, transformation to the sphere and then to the plane (by inversion) is adopted. The resulting plane problem is especially simple since the stress function is harmonic, not biharmonic as in the elastic problem.

The solution is in error in the neighborhood of the fixed edge of the dam, where the constraints exerted by the foundations induce a perturbation ("edge effect") involving bending stresses. The author illustrates these effects with a circular cylinder held rigidly at one end and subject to pressure. He suggests the adoption of a fixed edge following a trajectory of zero direct strain as a means of avoiding special reinforcement to cope with these bending effects.

This paper will assist the engineer by justifying economy in material and will permit him to approach the design of large single-span dams with greater confidence. The theory has application to other problems besides that of dams.

W. S. Hemp, England

82. J. Charles Rathbun and C. W. Cunningham, "Continuous frame analysis by elastic support action," *Proc. Soc. civ. Engrs.*, Apr. 1947, vol. 73, pp. 413–449.

The method of analysis developed contains features of both the slope-deflection and the moment-distribution methods. Properties of the conjugate beam are used to establish the relation between end-moments of members and their flexibility, defined as angular rotation of joints. The flexibility of intermediate supports is expressed by "elastic factors" representing the combined flexibilities of adjoining members and computed from equations of moment-equilibrium at the support.

Computation of elastic factors throughout the structure must begin at a support with known flexibility. If this condition cannot be met, certain joints are temporarily fixed against rotation and the solution later adjusted. Similarly, sidesway is only considered in a second step of approximation, in the form of shear requirements superimposed upon the first solution.

A. M. Freudenthal, U.S.A.

83. Robert Mayne, "Evaluation of various methods of rotor-blade analysis by means of a structural model," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 4, no. 2, pp. 62–73.

Results are presented for the experimental determination of bending moments in a structural model of a rotor blade. Only static type of flight loads are considered. Moments in the model are determined by multiplying the curvature by the bending stiffness EI. The curvature is obtained by numerical differentiation of experimentally determined slopes, which are obtained by using a transit and mirrors attached to the model.

A comparison with several theoretical solutions is made. The method of Cierva is found to be satisfactory for most design purposes. The "tabular method" (solution of finite difference equations) gives excellent agreement with experiment.

Stanley U. Benscoter, U.S.A.

84. O. C. Zienkiewicz, "The stress distribution in gravity dams," *J. Instn. civ. Engrs.*, Jan. 1947, vol. 27, no. 3, pp. 244–271.

Complete stresses computed by an approximate procedure are given for an actual dam profile and for a triangular profile with a vertical upstream face. The assumption is made that the material is homogeneous, elastic, isotropic, and continuous with a foundation of infinite extent consisting of the same material as the dam. The author does not consider the effects of "uplift" nor of expansion joints and cracks in the dam or foundation. Consequently the direct practical applications of the results may not be significant.

However, an abbreviated but intelligible outline of the method of calculation in an appendix is of interest. The procedure used is essentially a relaxation solution of equations developed from a finite difference analogue to the differential equation governing the Airy's stress function for plane stress or strain.

N. M. Newmark, U.S.A.

85. F. Panlilio, "The theory of limit design applied to magnesium alloy and aluminum alloy structures," *J. roy. aero. Soc.*, June 1947, vol. 51, no. 438, pp. 534–571.

Comparison is made between the allowable loads, producing stresses higher than the proportional limit, for structures made of material with a definite yield point and for those made of material which does not have a definite yield point, such as magnesium and aluminum alloy. Test results are given for single-span and two-span beams of the light alloys.

M. V. Barton, U.S.A.

Plastic Flow, Failure

(See also Revs. 36, 46, 51, 55, 56, 72, 85, 106, 114)

86. Julius Miklowitz, "The initiation and propagation of the plastic zone in a tension bar of mild steel under eccentric loading," *J. appl. Mech.*, Mar. 1947, vol. 14, no. 1, pp. 21–30.

In this paper a series of tension tests on flat bars of mild steel (cross section $^1/_8 \times 1^1/_2$ in.) are reported. Special gripping devices, by which the load could be applied with an eccentricity as much as $^3/_8$ in. from the prolonged center line of the specimen, were developed.

The yielding started as a wedge-shaped area at one edge of the bar. This wedge progressed across the bar and adjacent wedges formed as the bar was stretched. The formation and growth of these wedges are carefully described. The author observed that the yield load was greatest when the eccentricity was smallest, that there was a drop in load at the yield point for small but not for large eccentricities, and that the angle of the wedge was larger for the greater eccentricities.

An attempt was made to explain this behavior by taking into account such factors as the reduction in the eccentricity as yielding progressed, and the stress concentration at the boundary between the elastic and plastic regions.

Evan A. Davis, U.S.A.

87. N. Davidenkov, E. Shevandin, and F. Wittmann, "The influence of size on the brittle strength of steel," *J. appl. Mech.*, Mar. 1947, vol. 14, no. 1, pp. 63–67.

So-called "size effect" was first noted in 1912 by M. Charpy, who observed that in impact tests of similar notched bars the energy absorbed per unit volume decreased with increasing size. Such tests are characterized by the impact type of loading and by the high stress gradients due to the notch.

To investigate the effect of the impact loading, the authors made static bending tests of square notched specimens, more refined than similar tests made by Dragomirov in 1914. Plotting the load against bending deflection, the authors take the ratio of the maximum load to the load just preceding sudden drop in resistance, as a measure of the tendency to cold brittleness. The fact that this ratio falls from 1.4 to 1.1, as the specimen width rises from 10 to 20 mm, demonstrates that size effect is not confined to impact loading.

To check on the effect of large stress gradient, which, it has been argued, may make propagation of a brittle crack more difficult, the authors made impact tension and bending tests on unnotched round bars. The bars were of 0.25 carbon steel, annealed at 1000 C, to obtain large grains and brittle fractures at the low test temperatures. To prevent size effect from being obscured by lack of mechanical similarity of body forces, the weight of the hammer and the velocity were varied in proportion

to the specimen size. In all bending tests the radii of knife-edges and supports were also varied in this proportion.

Each test was run at a certain temperature which was raised from test to test at 5 C intervals, the lower limit of critical temperature being taken as that at which fractures were still consistently brittle, while the upper limit was taken as that at which tough fractures first occurred consistently. Plotting the specimen diameters against these temperatures, the average critical temperature was found to increase from −130 C for 2-mm-diam bars to −92 C for 10-mm bars in the tension tests, with similar results for the bending tests, showing that size effect is not confined to large stress gradients.

The authors then compare their test results with the statistical theory developed by W. Weibull ["A statistical theory of the strength of materials," *Roy. Swed. Inst. for Eng. Res.*, 1939, report no. 151] which predicts a "size effect" for brittle fracture even for simple static tension tests. They show that both the brittle strengths and the scatter found in their tests increase with a decrease in specimen size, as indicated by the theory. The authors conclude that the statistical theory predicts satisfactorily the influence of size on the brittle strength of steel.

A. Nadai, U.S.A.

88. Guido Oberti, "Research on initial stresses in structures (Ricerche sulle sollecitazioni preesistenti nelle strutture)," *G. Gen. civ.*, May 1947, vol. 85, no. 5, pp. 195–209.

A general review is presented of Mathar's, Sachs', and Tesar's methods of determining initial stresses by making slots or holes in the structures, with a theoretical and experimental analysis. The author prefers the method of making two opposite slots, and measures the deformation with mechanical or acoustical extensometers. He studies the distribution of stresses in the neighborhood of the slots by photoelasticity. For calibration he uses plaster blocks under uniform compression. Some applications to concrete and steel structures are described.

A. J. Durelli, U.S.A.

89. G. Hudson and M. Greenfield, "The speed of propagation of brittle cracks in steel," *J. appl. Phys.*, Apr. 1947, vol. 18, pp. 405–407.

Brittle failure was produced by a tensile test of 1-in-thick structural steel plate containing a sharp edge notch. The speed of propagation of the crack was measured and found to be about 40,400 in. per sec. The method consisted of recording the fracture of wires cemented across the expected path of the crack. Each wire on breaking introduced a step in the trace of an oscillograph. William N. Findley, U.S.A.

90. P. W. Bridgman, "The effect of hydrostatic pressure on the fracture of brittle substances," *J. appl. Phys.*, Feb. 1947, vol. 18, pp. 246–258.

Experiments are described in which a number of normally brittle materials are fractured under the action of hydrostatic pressures ranging up to 30,000 kg/cm² (425,000 psi) together with superposed tensile or compressive stresses. This work is a continuation of the author's original tests of 1912, in which he observed the so-called "pinching-off" effect, or "tensile fracture on planes of zero stress" caused by hydrostatic pressure on bars with free ends.

His most recent experiments were carried out on bars covered with a thin sheathing and with facilities provided for varying the end load. With superposed tension, Pyrex glass fractured when a net compressive stress existed on the plane of fracture, beryllium and phosphor bronze showed marked plastic deformation, Carboloy was brittle but the superposed tensile stress was three times the ordinary fracture stress, NaCl deformed plastically with no apparent change in optical homogeneity, and pipestone remained brittle with no improvement in tensile strength. With superposed compression, Pyrex glass failed by slipping on a plane at about 65 deg with the axis, at a superposed compressive stress of 47,000 kg/cm². Al_2O_3 single crystals slipped plastically on the basal plane without fracture.

The sheathing problem is discussed and its effect of causing fracture at superposed tensile-stress values numerically less than the hydrostatic pressure is considered. Fractures of brittle and ductile materials are treated from the point of view that energy release is one of the controlling factors. Griffith's conception of fracture as resulting from stress concentrations due to microscopic cracks is shown to be consistent qualitatively with experimental results. Louis F. Coffin, Jr., U.S.A.

91. C. L. Smith, "Thermal hardening of cadmium crystals," *Nature, London*, Oct. 4, 1947, vol. 160, no. 4066, pp. 466–467.

Orowan found that, after an annealing process, single crystals of chemically pure zinc show a higher critical yield stress than their predetermined value. The author finds that the phenomenon is also a property of single crystals of spectroscopically pure cadmium. His experiments show that a higher temperature anneal requires a shorter time to produce a certain increase in yield stress, and that the annealing period increases rapidly with the amount of prestrain given the crystal.

The main interest is centered about the influence of the crystal orientation. Under constant annealing conditions, values of the raised yield stress in shear were obtained for various values of the angle λ between the directions of the applied stress and the slip. The rise in yield stress was approximately zero at $\lambda = 40$ to 45 deg, but increased rapidly on both sides of this. The maximum was 70 per cent above the normal value. The author concludes that the orientation factor cannot be explained in the general terms of the movement of dislocations out of the crystal, a theory used to explain the other factors. Julius Miklowitz, U.S.A.

92. J. S. Koehler and F. Seitz, "Proposed experiments for further study of the mechanism of plastic deformation," *J. appl. Mech.*, Sept. 1947, vol. 14, no. 3, pp. 217–224.

The primary difficulty with the standard theory of dislocations in metals is connected with the initiation of the dislocations. It has always been necessary to assume ad hoc some sort of stress raisers. In the present discussion, this difficulty is avoided through the introduction of the assumption that dislocations are initially present along the mosaic boundaries. According to this concept the yield stress is that stress necessary to break the dislocations away from the mosaic boundaries.

Computations indicate that sufficient dislocations are already present in annealed metals to account for their capacity for plastic deformation. The experiments which would be required to confirm this concept are discussed. C. Zener, U.S.A.

93. J. G. Oldroyd, "A rational formulation of the equations of plastic flow for a Bingham Solid," *Proc. Camb. phil. Soc.*, Jan. 1947, vol. 43, part 1, pp. 100–105.

The author considers a material called a "Bingham Solid," which can support a finite stress elastically without flow and which flows with constant mobility when the stresses are sufficiently great. The material is considered isotropic and the strains are assumed small. Only deviatoric components of the stress and strain tensors are involved in the criterion for yield (tensor notation is used throughout).

It is shown that the principal directions associated with the yield stress tensor at a point in a region of plastic flow coincide with the principal directions of the strain tensor, and further that deviatoric components of the stress, rate of strain, and yield stress tensors are proportional; also that the Bingham Solid may

be regarded as a fluid with variable viscosity, a scalar function of the stress tensor, the scalar yield value, and the mobility. The author closes with the deduction that the plastic flow in a Bingham Solid occurs in such a manner that the rate of dissipation of energy is a minimum. Edward Saibel, U.S.A.

94. B. Gross, "On creep and relaxation," *J. appl. Phys.*, Feb. 1947, vol. 18, pp. 212–221.

The phenomenological theory for linear visco-elastic material is analyzed with the aid of Laplace transforms, Stieltjes' integral equation, and Heaviside operational calculus. Extensive reference is made to similar literature. Reversible effects only are considered; permanent deformations are not taken into account directly.

The usual distribution functions of retardation times of strain, and of relaxation times of stress, are represented as Laplace transforms of primitive functions. A general transformation formula is established between the primitive distribution functions, on the assumption that the principle of superposition is valid for creep and relaxation. Either numerical or analytic computation may be employed.

Application of the general formulas is made to a study of relaxation when creep is given by the frequently used simple power law. The results are then modified to give proper representation for a long as well as a short time. D. C. Drucker, U.S.A.

95. E. G. Thomsen, D. M. Cunningham, and J. E. Dorn, "Fracture of some aluminum alloys under combined stress," *Trans. Amer. Soc. mech. Engrs.*, Feb. 1947, vol. 69, no. 2, pp. 81–87.

By submitting thin-walled tubes to a combination of axial load and internal pressure, the authors investigate fracture criteria for the aluminum alloys 24S-T, 24S-T80, and 24S-T81, subject to biaxial stressing. The experimental technique includes the use of both tensile and compressive axial loads. A description of the test equipment is included.

The results indicate good agreement with predictions based on the critical shear stress law for fracture, although the planes of fracture do not always coincide with the expected direction of the plane of maximum shear stress. Martin Goland, U.S.A.

96. W. R. Dean and A. H. Wilson, "A note on the theory of dislocation in metals," *Proc. Camb. phil. Soc.*, Apr. 1947, vol. 43, part 2, pp. 205–212.

This note is to provide results for quantitative discussion of a dislocation theory of slip in crystals proposed by W. L. Bragg [*Trans. North East Coast Instn. of Engrs. and Shipbuilders*, 1945, vol. 62, pp. 25–34], which differs somewhat from G. I. Taylor's theory [*Proc. roy. Soc., London, Ser. A*, 1934, vol. 145, p. 362].

For this purpose stress and energy expressions are obtained for a dislocation effected by normal separation of the faces of a direct cut between two parallel holes in the infinite elastic solid. The density of such dislocations necessary to provide the maximum energy density which can be imparted to copper by cold work (which has been measured by Taylor and Quinney) is calculated according to Bragg's theory, and found to agree in order of magnitude with Bragg's own estimate. J. N. Goodier, U.S.A.

97. A. A. Markov, "Variation principles in the theory of plasticity," (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, May–June 1947, vol. 11, pp. 339–350.

The author proves three minimum principles concerning the velocity field in an incompressible, perfectly plastic solid, which obeys the stress-strain relation of von Mises. The simplest of these principles is concerned with the case where the velocities are prescribed on the surface, subject, of course, to the condition that the surface integral of the normal velocity component vanishes.

If the components of the velocity strain are denoted in the customary fashion by e_x, e_y, e_z, g_{xy}, g_{yz}, g_{zx} and if $E = [e_x^2 + e_y^2 + e_z^2 + (g_{xy}^2 + g_{yz}^2 + g_{zx}^2)/2]^{1/2}$, the minimum principle in question can be formulated as follows: amongst all velocity distributions which satisfy the boundary conditions and the equation of incompressibility, the actual distribution furnishes the smallest value for the volume integral of the invariant E.

The author's other minimum principles are concerned with the cases where there is given on the surface (*a*) the stresses, or (*b*) the tangential stresses and the normal velocity. The minimum principles are used to establish uniqueness of the stresses in the interior (to within an arbitrary constant hydrostatic pressure). The author assumes tacitly that the boundary conditions assure plastic behavior *throughout* the body, but in many practically important cases (e.g., indentation problems) this assumption is not justified. W. Prager, U.S.A.

98. W. Boas and R. W. K. Honeycombe, "The anisotropy of thermal expansion as a cause of deformation in metals and alloys," *Proc. roy. Soc., London, Ser. A*, Feb. 25, 1947, vol. 188, no. 1015, pp. 427–439.

The authors have previously reported that a cyclic heating and cooling of polycrystalline noncubic metals results in plastic deformation, as revealed by slip lines. The present paper describes an extension of their original work. The effects have been observed in zinc, tin, and cadmium, but in no cubic metal. It is inferred that the deformation arises from the lack of isotropy in thermal expansion of these noncubic metals, which enables changes in temperature to produce residual stresses.

One effect of this plastic deformation is the induction of grain growth in castings previously cooled to room temperature and then annealed. A second effect is inferred, namely, a gradual disintegration following repeated thermal cycles, in much the same manner as minerals of noncubic structure are known to disintegrate under such treatment. Clarence Zener, U.S.A.

99. G. H. Handelman, C. C. Lin, and W. Prager, "On the mechanical behavior of metals in the strain-hardening range," *Quart. appl. Math.*, Jan. 1947, vol. 4, pp. 397–407.

For the past thirty years the von Mises equations of flow have been used to describe the response of polycrystalline metals to stress in the plastic region. Systematic deviations from these equations have been observed in metals since the classic experiments of Lode in 1926. In recent years various attempts have been presented to generalize the original von Mises equations so as to correspond to the observed behavior. Such a generalization must of course take care of strain hardening.

This paper presents the most generalized form of the flow equations subject to certain assumptions regarding the material. These assumptions are essentially that (1) no creep occurs at constant stress, or conversely no stress relaxation at constant strain, (2) the increment of strain associated with an increment of stress is a function only of the stress and of the stress increment, and is independent of stress history, subject to the restriction of no unloading. The extent to which any actual metal satisfies these assumptions is not at present known.

Clarence Zener, U.S.A.

100. W. R. Osgood, "Combined-stress tests on 24S-T aluminum-alloy tubes," *J. appl. Mech.*, June 1947, vol. 14, no. 2, pp. 147–153.

Tests were made on tubes $1^3/_4$ in. ID, 0.05 in. thick, with ratios of hoop stress to axial stress of 0, $^1/_2$, 1, 2, and ∞. The thickness was chosen as thin as possible to minimize strain variations throughout the thickness, which were about $5^1/_2$ per cent in the circumferential direction as compared with 15 per cent in tests by other investigators. Great care was taken in machining a uni-

formly thin wall, in measuring thickness, and in devising a four-legged resistance-gage extensometer to measure diametral expansion and axial strains simultaneously almost up to rupture.

Results are presented in two plots: (*a*) maximum shearing stress vs. maximum shearing strain, and (*b*) octahedral shearing stress vs. octahedral shearing strain. About the same scatter (± 5 per cent on the stress axis) occurs on either plot, and is possibly due to anisotropy of material. No consistent change in shape of either plot accompanied the variation in ratio of circumferential to axial stress. Basic stress-strain relations for interpretation of observations in the tubes under combined stress are given. Thomas J. Dolan, U.S.A.

101. Julius Miklowitz, "The initiation and propagation of the plastic zone in a tension bar of mild steel as influenced by the speed of stretching and rigidity of testing machines," *J. appl. Mech.*, Mar. 1947, vol. 14, no. 1, pp. 31–38.

Constant strain rate tests on a low carbon steel and a 1 per cent silicon iron showed a pronounced speed effect. The upper and lower yield point and also the yield point elongation showed marked increases between the slowest head rate, 8.6×10^{-5} in. per min, and the fastest head rate, 15.4 in. per min. By carefully measuring the thickness and width of flat specimens the author was able to point out a plastic working length in the plastic region adjacent to the elastic region. Beyond this working length the plastic strain was constant and equal to the yield point elongation. The extent of this plastic working length varied directly as the speed of stretching.

The rigidity of the testing machine was varied by inserting helical springs in series with the specimens. One spring 9 times and one 30 times as flexible as the machine itself were used. For the most rigid machine the upper yield point was highest, but this was followed by a considerable drop in the load. For the softest spring the yield point was lowest and there was no drop in the load during the yield point elongation.

Evan A. Davis, U.S.A.

102. R. Hill, E. H. Lee, and S. J. Tupper, "The theory of wedge indentation of ductile materials," *Proc. roy. Soc., London, Ser. A*, Jan. 30, 1947, vol. 188, no. 1013, pp. 273–289.

A theoretical solution is obtained for the plane strain problem of plastic indentation of a triangular wedge into the plane boundary of a semi-infinite solid. The finite plastic deformation of the solid is taken into account. A St. Venant-Mises material is assumed, i.e., one which is rigid until a limiting stress condition is reached and (for plane strain) then flows under constant shear stress without change of volume. The wedge is considered to be frictionless and rigid and the plastic boundary to pass through its tip.

It is shown that the geometrical configuration is similar at every stage of the penetration so that it is not necessary to follow the history step by step. The complete stress and strain solution is obtained directly and a simple description is given of the motion of each point as the indentation proceeds. The equations of equilibrium and strain are referred to the slip lines and it is found that the field of slip lines consists of two regions of orthogonal straight lines with a fan region between them. The displaced portion of the originally plane surface becomes an inclined plane on each side of the wedge. Good agreement is shown between predictions of the theory and the actual deformation of a lead block.

A comparison is made between the average strain in an indentation test and in a tension test, with the object of showing the degree of approximation involved in the assumption of St. Venant-Mises plasticity.

Reference is made to the early work of Hencky and Prandtl but not to the more recent Russian developments of the theory of plasticity, e.g., Christianovitch on the method of characteristics and, more especially, Sokolovsky on indentation with various wedge and boundary shapes, [*J. appl. Math. and Mech., Moscow* (in Russian), 1940, vol. 4, no. 5–6, pp. 19–34], in which the change in the shape of the solid is not taken into account.

D. C. Drucker, U.S.A.

Design Factors, Meaning of Material Tests

(See also Revs. 75, 85, 90, 102)

103. E. P. Popov, "Correlation of tension creep tests with relaxation tests," *J. appl. Mech.*, June 1947, vol. 14, no. 2, pp. 135–142.

This paper gives a review of various creep strain-time-stress relations for simple tension, and shows how they can be applied to the tension creep-stress-relaxation problem in which the stress does not remain constant. The inadequacy of the commonly used methods, such as the log-log, semilog, and hyperbolic-sine creep rate-stress relations is noted. These methods neglect the primary stage of creep, and when they are used for predicting the stress relaxation-time relation under creep conditions, poor agreement with stress-relaxation test results is found.

Creep strain-stress-time relations which consider the primary stage of creep, such as the Soderberg method, are used by the author as a basis for predicting the creep tension-relaxation problem. Numerical and graphical procedures are explained for constructing stress relaxation-time curves based on the creep-time relations. The results obtained agree well with actual tension stress relaxation data for the material considered.

Joseph Marin, U.S.A.

104. Joseph Marin and F. B. Stulen, "A new fatigue strength-damping criterion for the design of resonant members," *J. appl. Mech.*, Sept. 1947, vol. 14, no. 3, pp. 209–212.

The authors present theoretical expressions for the resonant strength of cantilever beams under: (*a*) an internal energy absorption proportional to the third power of the stresses induced in the beam; (*b*) external viscous damping; and (*c*) a combination of (*a*) and (*b*). They propose a resonant-strength criterion for structural materials which depends on the fatigue strength, the damping characteristics, and the modulus of elasticity of the material. Alexander Yorgiadis, U.S.A.

Material Test Techniques

(See also Revs. 36, 44, 86, 89, 90, 95, 100, 101, 112, 118)

105. C. H. Desch, D. O. Sproule, and W. J. Dawson, "The detection of cracks in steel by means of supersonic waves," *Welding Res. Supplement*, Jan. 1947, vol. 12, no. 1, pp. 1–3.

This discusses developments in the use of sound waves to detect defects in metallic alloys and also to measure damping characteristics. The older method of detecting flaws by the change in "note" after striking the object with a hammer is limited, due to the wave length being large in comparison with the defect. To overcome these limitations the new technique uses "supersonic" or "ultrasonic" wave lengths of 1.6 cm to 0.6×10^{-6} cm. In steel these wave lengths become 5.1 mm for a 20,000-cycle frequency and 0.01 mm for a 5×10^6-cycle frequency. Wave lengths between these values are generally used.

The early German patents are cited together with the later developments in the USSR. The work in England in collaboration with the Hair Line Cracks Subcommittee of the Iron and Steel Institute, and the possibilities of the Sperry Supersonic Reflectoscope are mentioned. Frommer's method

of measuring damping capacity, using torsional methods, is also discussed.

The author concludes that the method does not seem to be of general application in the detection of material defects in steel because of its sensitiveness to variations in the structural condition. It may, however, form a valuable means of investigating changes brought about by heat-treatment. Application of the method to the examination of light alloys may be more successful since in these cases the structure is less dependent on the thermal treatment than in the case of steel.

John M. Lessells, U.S.A.

106. John R. Low, Jr., and Frank Garofalo, "Precision determination of stress-strain curves in the plastic range," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 4, no. 2, pp. 16–24.

Apparatus consisting of a ring dynamometer and a clip gage extensometer, both utilizing wire strain gages as indicators, was developed for obtaining tensile stress-strain diagrams of thin strips.

Tension tests were made to check the validity of a power function relation between true stress and natural strain. The stress-strain diagrams for six steels and two aluminum alloys had a true power function shape for strains between the point of yielding and that of maximum load. One steel, an 18-8 stainless sheet, did not conform to this shape. The authors attribute this deviation to the decomposition of the austenite during plastic deformation.

Evan A. Davis, U.S.A.

107. Victor Mathien, "The universality of the differential dilatometer with Chevenard photographic registration system (L'universalité du dilatomètre différentiel a enregistrement photographique système Chevenard)," *Rev. univ. Min.*, 1947, vol. 3, no. 7, pp. 259–266.

Two new uses of the Chevenard differential dilatometer are described: the calibration of electric thermocouples, and the composition control of silica bricks. The author introduces a few changes in the dilatometer for the latter purpose. The maximum absolute error in quantitative determination of silica components using the dilatometer is given as ±1.40 per cent. An example of application of the analysis method is given.

A. J. Durelli, U.S.A.

108. C. R. Amberg, "Transverse-strength machine based on chainomatic principle," *J. Amer. ceram. Soc.*, Aug. 1, 1947, vol. 30, no. 8, pp. 256–259.

The author describes an improved machine for determining the modulus of rupture of clay bars, similar to an earlier chainomatic machine designed and used by the ceramic department of the University of Illinois. Features of the improved machine are (1) greater range of load, (2) virtual freedom from vibration during application of load, and (3) ease of operation. Bars having modulus of rupture as low as 100 psi may be handled as readily as bars having modulus of 6000 psi. A damper attached to the lever system tends to smooth out variations in loading rate. Tests indicate that this machine gives more uniform results than the common lead-shot machine. Harry R. Neifert, U.S.A.

109. M. Greenfield and E. T. Habib, "High-speed compression tests on copper," *J. appl. Phys.*, July 1947, vol. 18, pp. 645–650.

An apparatus for obtaining dynamic compression tests on $^1/_2$-in-long cylinders at unit strain rates averaging about 1200 per sec is discussed, along with some test results obtained with this equipment on copper cylinders. The apparatus consists of a compressed-air gun which blows a hardened steel piston against the specimen. The energy absorbed in deforming the cylinder is determined by measuring the velocity of the piston before and after impact. Velocities are determined by making the piston break light beams through the gun barrel. These light pulses are transferred by photocells and an appropriate circuit to the vertical plates of an oscillograph.

A curve, obtained with copper specimens, is presented for energy per unit volume versus strain, from which a true stress-logarithmic strain curve is obtained. This is compared with a high-speed tension-stress curve obtained by Manjoine and Nadai [*Proc. Amer. Soc. Test. Mat.*, 1940, vol. 40, p. 822]. The stress to produce any strain is again found to be higher in a dynamic test than in a static test. The percentage increase in stress required to produce a given strain in copper dynamically, rather than statically, is about the same for compression and tension.

E. J. Ripling, U.S.A.

110. W. S. Erwin and G. M. Rassweiler, "Ultrasonic resonance applied to nondestructive testing," *Rev. sci. Instrum.*, Oct. 1947, vol. 18, no. 10, pp. 750–753.

The automatic sonigage consists of an electrical oscillator which supplies ultrasonic frequencies to a piezoelectric quartz crystal. The condenser of the oscillator is rotated at 1800 rpm, continuously varying the applied frequency over a given range, of which there are several. The electronic beam of a cathode-ray oscilloscope is synchronized with the rotation of the condenser. The quartz crystal is held against the surface of the material under test, setting up ultrasonic vibrations in the material. As the applied frequency passes through the fundamental or an harmonic frequency of the test member, there is an increase in the energy drawn by the crystal, which is indicated on the oscilloscope screen.

The application of this instrument to thickness measurements, flaw detection, etc., is carefully detailed. Interpretation of the screen patterns obtained during a test is discussed. Illustrations of typical patterns are given. George H. Lee, U.S.A.

111. Marcel Prot, "Fatigue test under progressive load (L'essai de fatigue sous charge progressive)," *C. R. Acad. Sci., Paris*, Oct. 20, 1947, vol. 225, no. 16, p. 669.

This paper proposes a more rapid and more accurate method for the determination of the endurance limit of materials. In place of the usual tests in which specimens are subjected to cycles of stress in which the minimum and maximum loads remain constant, tests are made in which the maximum load P increases linearly with time; that is, $P = \alpha t$, where α = a constant and t = the time. If for a given α, the load at rupture is P_r, then a plot between P_r and $\sqrt{\alpha}$ for various values of α gives approximately a straight line. The intersection of this straight line with the load axis gives a load value corresponding to the endurance limit of the material. Joseph Marin, U.S.A.

Mechanical Properties of Specific Materials

(See also Revs. 4, 43, 54, 87, 91, 95, 100, 106, 109, 182)

112. D. Telfair, C. H. Adams, and H. W. Mohrman, "Creep, long-time tensile and flexural fatigue properties of melamine, phenolic plastics," *Mod. Plastics*, May 1947, vol. 24, no. 9, pp. 151, 152, 236–248.

Data are reported for filled melamine and phenolic molded plastics at 25 C and 50 per cent relative humidity. Creep rates (and total creep) at 500 hours for these plastics, asbestos filled, were about two thirds of the values for the cellulose filled. The long-time tensile strength of cellulose-filled melamine was about twice that of wood-flour-filled phenolic, the respective values being 67 and 36 per cent of their short-time tensile strengths of 7300 psi for the melamine and 6100 psi for the phenolic.

Bending fatigue tests indicated the wood-flour-filled phenolic to have an endurance limit of 4000 psi (35 per cent of short-time flexural strength) and the cellulose-filled melamine an endurance limit of 3000 psi (31 per cent of short-time flexural strength). A theoretical discussion of possible differences in cross-linking of the molecules is presented as a probable explanation for differences in mechanical behavior of the two plastics.

Thomas J. Dolan, U.S.A.

113. Jay R. Burns, "Beryllium in magnesium casting alloys," *ATI Tech. Data Dig.*, Dec. 1, 1947, vol. 12, no. 11, pp. 5–12.

This work is primarily an investigation of oxidation control in magnesium foundry practice by small additions of beryllium. While beryllium affords some protection against burning, it also causes a marked grain coarsening with resultant decrease in physical properties. The conclusions are based on results of 14 experimental heats of AZ92 alloy, which is similar to Dow C and American Magnesium AM240 alloys.

J. F. Snider, U.S.A.

114. G. W. Washa, "Plastic flow of thin reinforced concrete slabs," *J. Amer. concr. Inst.*, Nov. 1947, vol. 19, no. 3, pp. 237–260.

New data on long-time tests of reinforced concrete slabs 3 in. thick are given, with particular consideration to the influence of slump, water-cement ratio, and curing methods. Some of the tests show an important increase in compressive strains and in deflections after five years of loading. An equation of the type $Y = Cx^{1/a}$ is suggested for the first part of the curve, expressing the relation between the plastic flow deflection Y in inches and the time x in days. The author gives the constants C and a for various slabs, and the limiting ages for which these constants are reasonably correct. A. J. Durelli, U.S.A.

115. C. A. Crawford, "Nickel-chromium alloys for gas-turbine service," *Trans. Amer. Soc. mech. Engrs.*, Aug. 1947, vol. 69, no. 6, pp. 609–612.

Data are presented on modulus of elasticity, Poisson's ratio, hardness, tensile properties, stress-rupture properties, creep resistance, fatigue resistance, ability to be fabricated, coefficient of expansion, thermal conductivity, specific heat, melting range, and other physical properties of a wrought alloy (Inconel X) and a cast alloy (50 Ni-20 Cr) for structural use at high temperature. J. D. Lubahn, U.S.A.

116. S. T. Collier, "Bond characteristics of commercial and prepared reinforcing bars," *J. Amer. concr. Inst.*, June 1947, vol. 18, no. 10, pp. 1125–1133.

Comparative values of bond resistance of $^7/_8$-in-diam reinforcing bars with "special" double-helical rib deformations, and with "commercial" transverse or longitudinal lug deformations are determined by pull-out tests. Although the "special" bars, in general, developed better bond resistance, the conclusiveness of the results appears to be affected by the fact that the position of casting is the most dominant single influence.

A. M. Freudenthal, U.S.A.

117. Howard Scott and R. B. Gordon, "Precipitation-hardened alloys for gas-turbine service. I.—Metallurgical considerations. II.—Design and application data," *Trans. Amer. Soc. mech. Engrs.*, Aug. 1947, vol. 69, no. 6, pp. 583–599.

The first part of this paper presents a detailed account of the metallurgical principles involved in the development of alloys for high-temperature service in gas turbines. Typical data for short-time tensile properties at 1000 to 1600 F are plotted. Four different alloys are discussed and detailed studies of aging, recrystallization effects, optimum heat-treatments, and precipitation hardening with titanium are presented.

The second part presents in the form of design curves creep data, total elongations, and time to rupture, for three different alloys at temperatures of 1200 F, 1350 F, and 1500 F. Values of fatigue strengths at these temperatures range from 52,000 to 80,000 psi. Other properties of these high-temperature alloys (thermal expansion, density, oxidation resistance) and their fabrication (forging, welding, machining) are discussed briefly.

T. J. Dolan, U.S.A.

118. J. T. Lapsley, A. E. Flanigan, W. F. Harper, and J. E. Dorn, "Room temperature tensile properties of aluminum-alloy sheet following brief elevated temperature exposure," *J. aero. Sci.*, Mar. 1947, vol. 14, pp. 148–154.

Some of the new high-strength materials show poor forming characteristics at room temperature, but show improved formability at elevated temperature. In general, the higher the temperature the greater the improvement. This paper gives the results of an investigation undertaken to evaluate the effects of short-time high-temperature exposure and forming operations, and to serve as a guide in the selection of permissible temperatures and times for hot-forming operations.

The tension test was used primarily to evaluate these permissible times and temperatures. The influence of temperatures and times of exposure on the yield strength, ultimate strength, and per cent elongation for various aluminum alloys are reported, for times ranging from five to twenty minutes and temperatures up to 500 F (260 C). Joseph Marin, U.S.A.

Mechanics of Forming and Cutting Processes

(See Revs. 49, 118)

Soil Mechanics; Seepage

(See also Rev. 168)

119. Ernst Maag, "Statical method for the calculation of bearing capacity of piles (Eine statische methode zur berechnung der tragfähigkeit von rammpfählen)," *Schweiz. Bauztg.*, July 26, 1947, vol. 65, no. 30, pp. 405–407.

A pile is conceived as a stepped foundation consisting of a finite or infinite number of cylinders stacked upon each other, to which the author applies equations derived by him for the carrying capacity of a foundation block in homogeneous soil. For a pile shaped like a truncated cone, the carrying capacity of the shaft portion is obtained by simple integration, and the carrying capacity of the bottom area of the cone is added. Form factors are given for other shapes. The equations are extended to cover heterogeneous subsoils and one example is given where the layers are fill, chalk, sand, and gravel. F. Hymans, U.S.A.

120. Reginald A. Barron, "Consolidation of fine-grained soils by drain wells," *Proc. Amer. Soc. civ. Engrs.*, June 1947, vol. 73, pp. 811–835.

Consolidation of fine-grained compressible soils subjected to new loads has been accelerated by drain wells. However, mathematical analyses of such wells have become available only recently, and are confined to wells having infinitely pervious well backfill and no peripheral smear. Complete analyses of consolidation by vertical and radial flow to well, for cases with or without peripheral smear and drain well resistance, are presented in this paper. These solutions should be regarded as approximate when applied to practical problems, because soils are not homogeneous, and because of incomplete knowledge of stress-strain consolidation characteristics of soils.

The author points out that most compressible soils are alluvial deposits and are more pervious in the direction of the bedding plane than in a perpendicular direction. When such soils are loaded, horizontal flow accelerates the consolidation of the soil mass as compared with strictly vertical flow. Vertical sand-filled drain wells are especially efficient in stratified soils because of the greater perviousness parallel to the bedding. Drain wells permit control over the water migrating in a horizontal direction, thus reducing the excess pore-water pressures that might be built up in the toe area of an earth fill.

Rollie G. Fehrman, U.S.A.

121. Ek-Khoo Tan, "Stability of soil slopes," *Proc. Amer. Soc. civ. Engrs.*, Jan. 1947, vol. 73, pp. 19–38.

Model tests of soil slopes were made to study the mechanics of failure in cases where the slide phenomenon is controlled by gravitational influences only. The angle of repose of cohesionless soil was found to be a superficial phenomenon, independent of the height of the slope. However, the inclination and the height of a bank of cohesive soil were found to depend upon cohesion. A technique to create an equivalent cohesion in a cohesionless soil was developed to produce actual sliding failures on a small-scale model, which were remarkably similar to those observed in nature.

Photoelastic studies were made on gelatin models of slopes, to obtain a clearer picture of the distribution and the relative magnitude of the shearing stresses set up within a slope, a method which seems to offer great possibilities. There was a remarkable resemblance between the pattern of shearing stress and the pattern of shearing strain obtained by this means and by the sand model. Failure seems to be initiated in a plastic region near the top of the slope.

An approximate mathematical solution of the slope problem based on the theory of plasticity was developed, to determine the possible position of the sliding curve and the stress conditions leading to failure. The theory shows the slip lines to be arcs of circles. The analysis shows that it is possible to have tensile stresses in the upper part of the slopes which might lead to tension cracks and final failure. A failure is imminent whenever the height and inclination of slope and the shearing properties of soil bear such a relation to each other that a plastic region develops.

The data in this paper indicate that failure within a soil mass results from plastic strain. Rollie G. Fehrman, U.S.A.

Potential Flow of Incompressible Fluids

(See also Revs. 135, 149, 155, 166)

122. G. F. Carrier, "Elbows for accelerated flow," *J. appl. Mech.*, June 1947, vol. 14, no. 2, pp. 108–112.

Numerous studies have been made of flow through channel bends in order to eliminate or reduce the tendency toward separation. Since separation occurs in regions of deceleration at a boundary, a desirable characteristic of such bends is a continuously increasing velocity. Although this condition is impossible for a bend of constant cross section, the author shows that it can be satisfied in two-dimensional flow if an over-all increase in velocity is permitted.

By an arbitrary assignment of the stream function and suitable conformal transformations, a family of elbow profiles is obtained, from which the one approaching the assumed asymptotic conditions most rapidly is selected. A complex function encountered in the solution is evaluated in tabular form to simplify applications. The method is extended to include flow of a compressible fluid, through use of the Kármán-Tsien approximate equation of state. John S. McNown, U.S.A.

123. Max Shiffman and D. C. Spencer, "The flow of an ideal incompressible fluid about a lens," *Quart. appl. Math.*, Oct. 1947, vol. 5, pp. 270–288.

The steady axially-symmetric potential flow associated with the motion of a body formed by the intersection of two spheres is studied. The potential function is constructed by an extension of the method of images. Using a toroidal co-ordinate system, multivalued dipoles are defined and their properties investigated.

Let the conventional single-valued dipole potential be $\varphi(\sigma,\psi,\psi_o)$ where σ, ψ are toroidal co-ordinates and ψ_o locates the singularity. Then one generalized dipole potential has the form

$$\Phi(\sigma,\psi,\psi_o) = \int \varphi(\sigma,\psi,\xi)/(\xi - \psi_o)d\xi$$

where the integration proceeds along a well defined contour in the complex ξ plane. Other dipoles are defined in an analogous fashion. As in the method of images, a series of these basic potentials is constructed in such a manner that the boundary conditions are satisfied for the class of obstacles given above.

The virtual mass of the fluid is evaluated and special cases discussed in some detail. In particular, for a symmetric lens, the derivative of the virtual mass with regard to a lens shape parameter is plotted. This would assist in sphere-fluid surface impact problem investigations. George Carrier, U.S.A.

124. R. C. Binder, "Calculation of diffuser efficiency for two-dimensional flow," *J. appl. Mech.*, Sept. 1947, vol. 14, no. 3, pp. 213–216.

In this paper the author presents a method for calculating the diffuser efficiency in two-dimensional incompressible flow without separation. The boundary-layer thickness is first calculated, which is a very tedious computation. Practically, separation usually occurs so that the method presented in this paper has only a limited application. The case when the disturbance widths from the two walls overlap each other is not presented in this paper.

No experimental data and computed values are given, but in general this is a good analysis of the flow through a two-dimensional diffuser. The author also has a new method for calculating the efficiency of a diffuser. P. C. Chu, U.S.A.

Turbulence, Boundary Layer, etc.

(See also Rev. 124)

125. G. B. Schubauer and H. K. Skramstad, "Laminar boundary-layer oscillations and stability of laminar flow," *J. aero. Sci.*, Feb. 1947, vol. 14, pp. 69–78.

In 1943 Schubauer and Skramstad reported upon an experimental project conducted for the National Advisory Committee for Aeronautics at the National Bureau of Standards (NACA Advanced Confidential Report, declassified) which served to verify beyond doubt the validity of the Tollmien-Schlichting theory of stability of laminary boundary-layer flow. This theory was thereafter considerably refined by Lin. The present paper contains a comprehensive review of the material covered in detail in the NACA report, and includes in addition a comparison with certain of Lin's results.

Detection of the predicted oscillatory disturbances in a laminar boundary layer became possible only after the turbulence of the experimental air stream was reduced to a level of 0.03 per cent, below that at which no further effect upon the location of the transition zone could be observed. Hot-wire oscillographs then revealed regular fluctuations within the laminar layer, which rapidly grew in magnitude and culminated in intermittent bursts just ahead of the transition zone, regardless of whether or not the actual transition was eliminated by producing locally a negative

pressure gradient. Analysis of the oscillograph records provided values which all fell along one branch of the theoretical curve.

Controlled frequency studies were then undertaken through use of a very fine electrically excited ribbon vibrator placed within the boundary layer, which permitted systematic investigation of damping and amplification of oscillations and complete definition of the neutral-oscillation curve for zero pressure gradient. Variation of the distance between the vibrating ribbon and the point of measurement yielded results in agreement with the predicted velocity of the neutral waves, and simultaneous measurements with two hot-wire instruments at the same section checked the theoretical indication of phase reversal with distance from the boundary. Supplementary tests in streams with positive and negative pressure gradients showed, as was to be expected, that these would have an additional amplifying or damping effect, respectively, if sufficiently large.

Hunter Rouse, U.S.A.

126. G. F. Carrier, "The boundary layer in a corner," *Quart. appl. Math.*, Jan. 1947, vol. 4, pp. 367–370.

An analysis is given of the laminar flow of an incompressible, low-viscosity fluid which impinges with a uniform velocity on the edges $x = 0$ of the half planes $y = 0$, $z = 0$. A solution of the Navier-Stokes and continuity equations is put in a series form, and the numerical evaluation carried out by the relaxation method. R. C. Binder, U.S.A.

127. P. Y. Chou, "The turbulent flow along a semi-infinite plate," *Quart. appl. Math.*, Oct. 1947, vol. 5, pp. 346–353.

This paper shows that von Kármán's logarithmic law of velocity distribution is consistent with the author's analysis of turbulent flow reported in an earlier paper [*Quart. appl. Math.*, 1945, vol. 3, pp. 38–54]. Simplifications and assumptions made include dynamical similarity downstream, Reynolds' equation for mean motion, equations of double and triple correlations, and constancy of microscale of turbulence and of the shearing stress across the boundary layer.

The same equations are also applicable to the temperature distribution of the boundary layer of a heated plate. The analogy to the velocity distribution in a channel, as treated by the author [*Quart. appl. Math.*, 1945, vol. 3, pp. 198–209], is shown and is used as reference. H. P. Liepman, U.S.A.

128. A. A. Townsend, "Measurements in the turbulent wake of a cylinder," *Proc. roy. Soc., London, Ser. A*, Sept. 9, 1947, vol. 190, no. 1023, pp. 551–561.

A hot wire anemometer was used to study the fully developed wake behind a cylinder in a low-turbulence air tunnel. Intensity measurements at Reynolds numbers from 100 to 7000 indicated essentially similar turbulence distributions sufficiently far downstream. Comprehensive measurements were made with a 1.12-mm cylinder at speeds of 560 and 1120 cm per sec for downstream distances up to 1000 diameters.

The intensities of the turbulent velocity components were nearly equal except in the wake center where the component at right angles to the flow and cylinder axis was considerably greater. The velocity fluctuations were normally distributed except near the edge of the wake where they were skewed. The derivatives in the downstream direction of the longitudinal and transverse correlations, obtained from the time differential of the velocity fluctuation, were nearly constant across the wake, with the transverse one about twice the longitudinal as in isotropic turbulence.

The kinetic energy of turbulence at various cross sections in the wake was calculated from the turbulence intensities and found to be initially larger than the kinetic energy of the mean flow. It decreased at a faster rate, until equilibrium seemed to be reached for distances in excess of 1000 diameters, with the mean flow energy 4.5 times that of the turbulence.

J. M. Robertson, U.S.A.

129. G. B. Schubauer and H. K. Skramstad, "Laminar boundary-layer oscillations and transition on a flat plate," *J. Res., Nat. Bur. Stands.*, Feb. 1947, vol. 38, no. 2, pp. 251–292.

This paper reports an investigation in which oscillations were discovered in the laminar boundary layer along a flat plate. These oscillations were found during the course of an experiment in which transition from laminar to turbulent flow was being studied on the plate, as the turbulence in the wind stream was being reduced to unusually low values by means of damping screens.

The first part of the paper deals with experimental methods and apparatus, measurements of turbulence and sound, and studies of transition. A description is then given of the manner in which oscillations were discovered and how they were found to be related to transition, and then how controlled oscillations were produced and studied in detail.

The oscillations are shown to be the velocity variations accompanying a wave motion in the boundary, this wave motion having all of the characteristics predicted by a stability theory based on the exponential growth of small disturbances. A review of this theory is given. This work is thus experimental confirmation of a mathematical theory of stability which has been in the process of development for a period of approximately 40 years, mainly by German investigators. R. C. Binder, U.S.A.

Compressible Flow, Gas Dynamics

(See also Revs. 122, 151, 153, 156, 158, 160, 164, 180, 181, 183, 184, 186)

130. M. H. Martin, "A problem in the propagation of shock," *Quart. appl. Math.*, Jan. 1947, vol. 4, pp. 330–348.

This paper is concerned with the calculation of the behavior of a gas, starting from the initial condition that its density has two different (constant) values in two adjacent regions of space. It is restricted to the case of one space variable, a very small change in density, and (in the latter portion) a monatomic gas. The behavior studied is taken to be an idealization of that of the atmosphere following an explosion within a long cylinder.

The paper shows that the two initial shocks give rise to shocks traveling in opposite directions, at first with a constant supersonic velocity and then with a continuously decreasing velocity, approaching the velocity of sound. The behavior of the gas is followed only up to the point where the mapping of the Riemann (r, s) plane on the physical (x, t) plane loses its one-to-one character. Two pages of figures, showing the qualitative variation of density with both distance and time, are included.

Charles Concordia, U.S.A.

131. Yung-Huai Kuo, "The propagation of a spherical or a cylindrical wave of finite amplitude and the production of shock waves," *Quart. appl. Math.*, Jan. 1947, vol. 4, pp. 349–360.

This paper deals with the question whether and when an isentropic, compressible flow of cylindrical or spherical symmetry will generate a shock wave. The question arises because of the interaction between the tendency of the front of a wave of finite amplitude to become steeper and the decrease of energy intensity due to the lengthening of the wave front while proceeding.

The author, neglecting viscosity and heat conduction, shows that a "limiting line" can appear. At this limiting line velocity gradient and pressure gradient become infinite, which is equivalent to the occurrence of a shock wave. However, for the entire

field of flow, no actual solutions of the problem can be given since the continuation of the flow downstream of the limiting line is found to be not possible, and any continuation tried leads to a cuspidal "turning back of the solution." A removal of this difficulty is attempted by a generalization of the equation of change of state in the original system of equations.

The methods of this paper use the theory of characteristics applied to the original system of equations, with distance and time as independent variables, and flow velocity and local sound velocity as dependent variables. This theory is also applied to the system of equations transformed by the interchange of independent and dependent variables. The criterion of the appearance of the shock wave is shown to be the vanishing of the value of the denominator determinant (Jacobian) of the transformations formulas. The curve on which this happens in the time-distance plane is a limiting line provided that a certain factor of the equation of characteristics remains finite. In the case of the "lost solution," where the Jacobian is identically zero, a limiting line is also found. H. J. Reissner, U.S.A.

132. Jean Villey, "On the position of the sonic section in the De Laval nozzle (Sur la position de la section sonique dans la tuyère De Laval)," *C. R. Acad. Sci., Paris*, Jan.–June 1947, vol. 224, pp. 27–29.

The sonic section in a De Laval nozzle is discussed for the case when the gas flow is linear and nonisentropic, by the application of logarithmic derivatives. It is found that, in general, the sonic section will not be at the narrowest section (throat) of the nozzle. In order to have the sonic section at the throat, a relation between heat added per unit mass and the change of kinetic energy due to friction must be satisfied. Application to nozzles of circular cross section is discussed.

Ahmed D. Kafadar, U.S.A.

133. Theodore von Kármán, "Supersonic aerodynamics—principles and applications," *J. aero. Sci.*, July 1947, vol. 14, no. 7, pp. 373–402.

The aim of the author seems to be a survey of the results of investigations in supersonic aerodynamics, conveying a clear physical picture, but without the mathematical details which would conceal the gist of the problem. The basic characteristics of supersonic flow are condensed into three statements: (*a*) "The rule of forbidden signals," (*b*) "Zone of action and zone of silence," (*c*) "The rule of concentrated action."

The mechanism of drag is then explained with the aid of momentum flux. Assuming small disturbances from the main stream, the author illustrates the different situation as regards drag production between incompressible flow, subsonic flow, and supersonic flow. The wave drag in supersonic flow is connected with the rule of concentrated action.

Pursuing further the small disturbance, or linearized theory of drag, the author discusses the wave drag in two-dimensional flow and the wave drag of a body of revolution. For airplane wings two methods are presented for drag calculation: the method of source and sink distribution and the method of Fourier integrals. The latter method is applied to calculate the wave drag of swept wings and arrowhead wings. The results of the calculations are shown in numerous graphs, together with different approximating formulas for different regions of the Mach number M.

The author then points out that the drag of the wing, according to the linear theory, is independent of the flight direction, whether forward or backward. The spanwise drag distribution is, however, different. For sweptback wings, the total drag acts on the center section, whereas in the case of sweep forward, a considerable portion of the drag acts on the tips.

The question of lift is taken up first with an explanation of its mechanism. The essential difference in the horseshoe-vortex field behind the wing for subsonic and supersonic flows is shown. The author also shows the balance of lift force and the fluid pressure on the ground and on the control surface. There is an essential difference here also for supersonic flow and for subsonic flow. The drag due to a given lift distribution is then treated by the method of Fourier integrals.

The author then discusses the separation of leading edge and trailing edge into supersonic and subsonic regions and the problem of choice of airfoils for the different locations of a swept wing. The delta wing is the next problem investigated.

The author then departs from the linearized theory, and discusses various aspects of modification when high-order effects are considered, such as detached shock wave, and the upper and the lower critical Mach number of transonic flows over a body. This is followed by a section on the characteristics of transonic flows, relating the following similarity formulas for drag coefficient C_D and lift coefficient C_L of a thin airfoil

$$C_D M^2 \Big/ \left(\frac{t}{c}\right)^{5/2} = \text{a function of } \frac{\left(\frac{t}{c}\right)^{1/2}}{\sqrt{1-M^2}},$$

$$C_L M^2 / i^{2/3} = \text{a function of } \frac{i^{1/3}}{\sqrt{1-M^2}}$$

where t/c is the thickness ratio and i the angle of attack of the airfoil.

The survey ends with a method for predicting the range of supersonic airplanes. H. S. Tsien, U.S.A.

134. John W. Tukey, "Linearization of solutions in supersonic flow," *Quart. appl. Math.*, Oct. 1947, vol. 5, pp. 361–365.

It is suggested that in some cases the "linearization of solutions" for problems of supersonic flow might be more convenient than the "linearization of equations." An empirical linearized equation for supersonic flow over cones is presented, which gives agreement on pressure coefficients to within about 5 per cent of the exact solution. It is proposed that empirical linearizations for drag, lift, etc., in supersonic flow be of the form $Ap + Bq$ where p denotes the pressure, q the dynamic head, and A and B are constants. Ascher H. Shapiro, U.S.A.

135. E. R. Van Driest, "Streamlines for the subsonic flow of a compressible fluid past a sphere," *J. appl. Phys.*, Feb. 1947, vol. 18, pp. 194–198.

This paper presents the results of calculations of the streamlines for the irrotational flow of a compressible perfect fluid past a sphere, for a Mach number M (for the undisturbed stream) of 0.5, and compares these results to those for the case of an incompressible fluid. The calculations are based on the second approximation to the velocity potential, expanded in powers of M,[2] as previously given by Tamada. Velocity and pressure comparisons are included. Charles Concordia, U.S.A.

136. H. Bondi, "Waves on the surface of a compressible liquid," *Proc. Camb. phil. Soc.*, Jan. 1947, vol. 43, part 1, pp. 75–95.

The paper is an unusually well organized and lucid exposition of the subject. The author summarizes his work as follows:

"The possibility of the propagation of disturbances along a free surface with a velocity exceeding the velocity of sound in the liquid is examined. Simple necessary conditions for the occurrence of such ultrasonic propagation are found in terms of the boundary condition. It is shown that the conditions are not satisfied in the case of gravity waves, and a detailed examination is made of the pressure and surface waves due to a localized dis-

turbance. Capillary waves are shown to satisfy the conditions for ultrasonic propagation, the existence of a 'faster-than-sound' wave is proved, and its main properties are examined."

The main property of the capillary ultrasonic waves is shown to be that they have a wave length of molecular dimensions and so lack physical significance. In addition to this element of theoretical interest, the author makes a valuable contribution to the practical problem of determining the characteristics of both gravity and capillary waves set up by a localized subsurface disturbance. F. Everett Reed, U.S.A.

137. Jean Villey, "Subsonic flows in a De Laval nozzle (Les écoulements subsoniques dans une tuyère De Laval)," *C. R. Acad. Sci., Paris*, Jan.–June 1947, vol. 224, pp. 329–330.

This is a complement to a previous paper by the author [*C. R. Acad. Sci., Paris*, Jan.–June 1947, vol. 224, pp. 27–29]. Linear nonisentropic flow in a De Laval nozzle when the back pressure is above the critical (subsonic flow) is discussed. It is pointed out that under these conditions maximum velocity and specific volume, and minimum temperature and pressure, will not occur in the same section or at the throat. Locations of these maxima and minima are discussed. Ahmed D. Kafadar, U.S.A.

138. Ascher H. Shapiro and Gilbert M. Edelman, "Method of characteristics for two-dimensional supersonic flow—graphical and numerical procedures," *J. appl. Mech.*, June 1947, vol. 14, no. 2, pp. 154–162.

The method of characteristics for two-dimensional supersonic flow is reviewed and summarized. A characteristics chart is presented for use with the graphical procedure of Prandtl and Busemann. A new numerical procedure is described which eliminates graphical operations and which allows of more accurate solutions. To facilitate the numerical method, a table of useful functions is included.

The methods described are accurate to second order terms. They are, however, those which require a fixed characteristic net in the hodograph plane. The iteration procedure (numerical or graphical-numerical) which applies to more general cases is not discussed. C. C. Lin, U.S.A.

139. Lester L. Cronvich, "Numerical-graphical methods of characteristics for plane potential shock-free flow problems," *J. aero. Sci.*, Apr. 1947, vol. 14, pp. 237–242.

Applications of the method of characteristics to problems in two-dimensional supersonic flow are discussed. A mathematical introduction is included, but seems somewhat incomplete. In addition to the conventional "field" method for numerical and graphical solution, a "lattice-point" method is presented. Complete step-by-step details for employing both methods are given, with examples of each. The "field" and "lattice-point" methods are compared with each other and their relative usefulness for different types of problems is appraised. The examples include the flow in a hyperbolic channel and the design of nozzle contours to provide a uniform, parallel flow. Ascher H. Shapiro, U.S.A.

140. Hsue-shen Tsien, "Flow conditions near the intersection of a shock wave with solid boundary," *J. Math. Phys.*, Apr. 1947, vol. 26, no. 1, pp. 69–75.

This is an extension of an investigation by Emmons [*Nat. Advis. Com. for Aero., Tech. Note No. 1003*, 1946 (Appendix 2)], concerning plane flow of a frictionless fluid past a solid boundary, where a shock wave divides the flow into supersonic (upstream) and subsonic regions. Since the boundary has continuous slope, the wave intersects it at a right angle. Moreover, the wave is assumed to be plane. Approximately, for locations close to the boundary, normal-shock relations can be used. The surface may have a discontinuity of curvature at the wave.

The result is a relation between Mach numbers before and after the shock wave and the respective radii of curvature. This is the same relation as was found by Emmons without considering the pressure gradients that may exist here along the solid boundary. The ratio of pressure gradients is related to the Mach numbers, the ratio being always negative.

In the reviewer's opinion, the author's justification for assuming a plane shock, namely that "the effect of any finite curvature of the shock will be of second order," requires further study.

W. R. Sears, U.S.A.

141. W. R. Sears, "A second note on compressible flow about bodies of revolution," *Quart. appl. Math.*, Apr. 1947, vol. 5, pp. 89–91.

This is a correction of the author's previous Note [*Quart. appl. Math.*, 1946, vol. 4, pp. 191–193] on the linearized subsonic flow over a slender body of revolution. If u is the maximum velocity on the surface of the body, the author arrives at the correction formula for the disturbance velocity, $(u - U)/U = \beta^{-2} F(\beta n)$, where U is the free-stream velocity, $\beta^2 = 1 - M^2$, M is the free-stream Mach number, and $F(n)$ is the maximum superstream velocity ratio in incompressible flow, n being the thickness ratio.

Y. H. Kuo, U.S.A.

142. Edmund V. Laitone, "Exact and approximate solutions of two-dimensional oblique shock flow," *J. aero. Sci.*, Jan. 1947, vol. 14, pp. 25–41.

This paper deals with the practical solution of two-dimensional supersonic air flow problems involving oblique shock waves. The basic ideas and usual formulas for isentropic flow, oblique shock waves, and normal shock waves are derived in succession. Their use is made convenient by graphical presentation. Various power series approximations to these formulas are presented and their degree of approximation is shown graphically.

Some of these approximations have appeared in previous literature, while others are new in this paper. Because of page space available, the graphical results are too small to be of much direct use in computations. The paper is most useful for its collection of shock equations and the quantitative discussion of the various approximations. Howard W. Emmons, U.S.A.

143. B. L. Hicks, P. E. Guenther, and R. H. Wasserman, "New formulations of the equations for compressible flow," *Quart. appl. Math.*, Oct. 1947, vol. 5, pp. 357–361.

The compressible flow equations are formulated for use in the analysis of combustion problems. The authors recommend the use of Mach and Crocco vectors as a simplification over the velocity vector in the equations of motion and continuity. The Mach number M is the ratio of fluid velocity to the local velocity of sound. The less familiar Crocco number W is the ratio of the fluid velocity to the maximum possible fluid velocity (the latter being a function of the stagnation temperature).

The equation of motion is expressed in terms of either the Mach or Crocco vector together with the pressure, and the continuity equation is given in terms of either vector. Examples are given which lead to some generalized conclusions.

Charles A. Meyer, U.S.A.

144. N. P. Bailey, "Abrupt energy transformation in flowing gases," *Trans. Amer. Soc. mech. Engrs.*, Oct. 1947, vol. 69, no. 7, pp. 749–763.

Experimental pressure measurements are presented for the supersonic flow in straight tubes, in free jets, and combustion flow in tubes, while measurements are given for temperature variations in a free jet of high-temperature combustion gases.

The experimental three-dimensional shock flow is analyzed on the basis of two-dimensional plane-shock theory. In the reviewer's opinion, this can lead to erroneous conclusions because additional flow compression must occur behind conical shocks in

order to satisfy the continuity relations. Also, the low Reynolds number can prevent the occurrence of strong shocks. At low Reynolds number the boundary layer thickens rapidly or actually separates from the walls, so the supersonic flow is compressed to a Mach number near unity before any shocks appear. The shock waves which finally compress the flow to a subsonic Mach number are then relatively weak, with practically no total pressure loss. This explains the experimental behavior, where the flow fluctuates and is not full because of the variation of the thickened boundary layer. E. V. Laitone, U.S.A.

145. Philip Rudnick, "Momentum relations in propulsive ducts," *J. aero. Sci.*, Sept. 1947, vol. 14, pp. 540–544.

This paper presents an analytical approach which is useful in studies of the forces developed by the flow of a gas in a duct. The analysis is for steady, one-dimensional, compressible flow. A function called "stream thrust" is defined as $A(p + \rho V^2)$ where A is area of cross section, p is pressure, ρ is density, and V is velocity. Stream thrust has the fundamental property that the difference between its values at any two sections of a stream gives the resultant of all normal and tangential forces acting on the duct walls between these sections.

A description of the function is followed by some illustrations of its usefulness in connection with supersonic ramjet design. The function is applicable to reversible processes and to irreversible processes involving friction and shock. The concept of conservation of stream thrust is analogous to the concepts of conservation of mass, conservation of energy, and conservation of entropy.

R. C. Binder, U.S.A.

146. Stefan Bergman and Leonard Greenstone, "Numerical determination by use of special computational devices of an integral operator in the theory of compressible fluids. 1. Determination of the coefficients of the integral operator by the use of punch card machines," *J. Math. Phys.*, Apr. 1947, vol. 26, no. 1, pp. 1–9.

The author has previously shown [*Trans. American Mathematical Society*, 1945, vol. 57, pp. 299–331] that the stream function $\psi(\lambda, \theta)$ expressed in terms of the speed parameter λ and the inclination θ of the velocity vector can be obtained as the imaginary part of a series. Each term of the series is a product of a universal function $Q^{(n)}(2\lambda)$ and a function $g^{[n]}(Z)$ of the variable $Z = \lambda - i\theta$. The present paper is concerned with the method of the numerical computation of $Q^{(n)}$. The method developed involves only integration for odd n. Since the integrations can be carried out in closed form for $Q^{(1)}$, $Q^{(3)}$, only three numerical integrations are needed for the determination of the first ten $Q^{(n)}$.

H. S. Tsien, U.S.A.

147. T. Y. Thomas, "On curved shock waves," *J. Math. Phys.*, Apr. 1947, vol. 26, no. 1, pp. 62–68.

Suppose in a steady motion of a frictionless gas there exists a curved discontinuous surface, namely a shock wave which divides the flow field into two regions. If the motion in front of the shock and the orientation thereof are known, the velocity u_α, pressure p and density ρ immediately after the shock are completely determined by the Rankine-Hugoniot relations.

The main object of this paper is to proceed further in obtaining the derivatives $u_{\alpha,\beta}$, $p_{,\alpha}$ and $\rho_{,\alpha}$ (using indicial notation, the comma signifying partial differentiation) after the shock, by making use of the Eulerian equations, equations of continuity and the constancy of the entropy along a stream line. As the author states, the essential contribution of the paper then consists in devising a technical procedure by which these derivatives are easily obtained, and the formulas for them appear as rather compact expressions. In the case of two-dimensional motion with uniform flow in front of the shock, explicit formulas are presented. Y. H. Kuo, U.S.A.

148. Gottfried Guderley, "New aspects of transonic flow theory," *ATI Tech. Data Dig.*, Nov. 1, 1947, vol. 12, no. 9, pp. 5–18.

The article seeks to show that the transition of the flow pattern from the subsonic to the supersonic state is probably continuous. Only the physical aspects of the phenomena are considered.

The case of the De Laval nozzle is discussed, where the incoming flow is subsonic and changes to supersonic at the restriction in the nozzle. Another example is the transition from pure to mixed flow patterns, as in the supersonic flow over a wedge. This results in a subsonic field included in the supersonic field. This flow is extended to the airfoil with a blunt or sharp leading edge.

The effect of changing the contour of a body from that of a known stream line is considered by investigating the effect of a protuberance in the wall of a cylinder with subsonic and with supersonic parallel flow. The paper describes the flow over a profile and the effect of the trailing edge on the nose of the airfoil at various Mach numbers. A reason for instability of the flow patterns was not found, at least for flow around a profile.

Nicholas Di Pinto, U.S.A.

Aerodynamics of Flight; Wind Resistance

(See also Revs. 7, 133, 161, 164, 180)

149. Herbert S. Ribner, "The ring airfoil in nonaxial flow," *J. aero. Sci.*, Sept. 1947, vol. 14, pp. 529–530.

The lift on a ring airfoil inclined to the axis of flow is analyzed by the lifting-line theory. It is first assumed and then proved that the fluid passing through the ring is deflected as a rigid body through an angle less than the angle of attack for large aspect ratios.

An extension of the theory to include very low aspect ratios for which the angles of deflection and attack are equal is shown to yield half as great a lift as in the former case. In each instance the lift is twice that for a flat elliptical plate of span equal to the diameter and having one quarter the area of the ring; the downwash and ratio of lift to induced drag are the same for both.

John S. McNown, U.S.A.

150. John K. Northrop, "The development of all-wing aircraft," *Engineering*, July 11, 1947, vol. 164, no. 4250, pp. 45–48.

The paper traces the development of the flying-wing-type airplane at Northrop, and includes an elementary discussion of the problems and potentialities of this type in relation to conventional machines. The author feels that the use of boundary layer control in conjunction with turbojets or turboprops will make possible higher maximum lift coefficients, this in turn making possible smaller wing area and a fuller capitalization of the low parasite drags. At such time the author predicts cruising ranges 158 to 130 per cent of comparable conventional types, and maximum speeds of 124 to 114 per cent of the same.

C. B. Smith, U.S.A.

151. A. E. Puckett and H. J. Stewart, "Aerodynamic performance of delta wings at supersonic speeds," *J. aero. Sci.*, Oct. 1947, vol. 14, no. 10, pp. 567–578.

Previously developed methods of solving linearized supersonic-flow equations, the source-superposition method of A. E. Puckett ["Supersonic wave drag of thin airfoils," *J. aero. Sci.*, Sept. 1946, vol. 13, no. 9, pp. 475–484] and the conical-flow-conformal-mapping method of H. J. Stewart ["The lift of a delta wing at supersonic speeds," *Quart. appl. Math.*, Oct. 1946, vol. 4, no. 3] are used to compute "linearized" drag, lift, and moment coefficients of delta wings. Here delta wings refer to a wide family of triangular wings with double-wedge airfoil sections, infinite taper ratio, and almost arbitrary sweep of leading and trailing edges and of the line of maximum thickness.

Results are presented in the form of twelve charts and several

tables. These are primarily intended for the designer, but should also aid in an understanding of three-dimensional supersonic flows in general. An approximate theory of "the leading-edge thrust" is included. M. V. Morkovin, U.S.A.

152. W. Tye, "Gusts," *J. roy. aero. Soc.*, Sept. 1947, vol. 51, no. 441, pp. 721–732.

This is a summary, largely qualitative, of present knowledge on the subject of atmospheric gusts and their effects on airplanes. The author discusses the occurrence of gusts in the atmosphere, their characteristics, effects on flexible wings, loss of control in gusts, spanwise distribution of gust loads, and a suggested rational type of airplane design requirement. He states that his data are from unpublished reports of various governmental agencies. W. R. Sears, U.S.A.

153. Frank J. Malina and Martin Summerfield, "The problem of escape from the earth by rocket," *J. aero. Sci.*, Aug. 1947, vol. 14, pp. 471–480.

The escape problem is reconsidered in the light of propellants available as a result of wartime development. The pertinent characteristics of these propellants are given and the resulting performance of several types of rockets estimated. It is shown that with the propellants now available it is not feasible to design a single-stage rocket capable of escape. Escape with a multi-staged rocket capable of carrying a small pay load does, however, appear possible. It is estimated that a 10-stage acid-aniline rocket of 70-ft length and 7-ft diameter could escape with a 10-lb pay load.

In general the effects of the principal design variables are shown, and useful estimates based on recent experience of such parameters as the required structural weight to gross weight ratios are given. C. B. Smith, U.S.A.

154. William F. Milliken, Jr., "Progress in dynamic stability and control research," *J. aero. Sci.*, Sept. 1947, vol. 14, pp. 493–519.

The author summarizes the need for, and objectives of research in this field, and outlines a broad research program toward these ends. For longitudinal motion a new method for obtaining quantitative dynamic data in flight, involving the use of a modified automatic pilot to oscillate the elevator sinusoidally, is described in detail. From the airplane response, the stability derivates are evaluated by known methods of analysis. This method was applied to a B-25 J airplane and the results are compared with those from conventional theory.

Instrument difficulties encountered in these tests and special equipment developed for them are described. Possibilities of this method and of similar methods with different forcing functions are explored, and the extension to the lateral stability problem is treated briefly. A synopsis of previous dynamic research and a bibliography of 91 references are included. J. M. Wild, U.S.A.

155. Dimitri Riabouchinsky, "Some remarks concerning the behavior of aspiration slots (Quelques remarques sur le procédé des fentes d'aspiration)," *C. R. Acad. Sci., Paris*, July–Dec. 1947, vol. 225, pp. 87–89.

This brief note is an elaboration of studies made by the author and others beginning as far back as 1924. The problem deals with the two-dimensional ideal fluid motion around bodies incorporating so-called "source-vortices," which are considered as a means of representing the effect of aspirating slots. The methods of conformal transformation are applied, using functions proposed by Tchapliguine and by von Kármán and Trefftz, the latter being the basis for development of air foil sections with crescent-shaped cores. Lift and drag forces are calculated and an application to a wing propelled by a built-in rocket is suggested.

M. J. Thompson, U.S.A.

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 17, 25, 27)

156. John W. Miles, "The aerodynamic forces on an oscillating airfoil at supersonic speeds," *J. aero. Sci.*, June 1947, vol. 14, pp. 351–358.

The author assumes the conditions for a linear, supersonic flow with free-stream velocity U in the direction of the positive x-axis. The velocity potential of the flow is expressed in the form of an integral, representing at each point the potential of a distribution of sources over the airfoil. The strengths of the sources vary harmonically with the time t, and the integration is extended over only those sources lying within the forward half of the Mach cone extending upstream from the point. By placing the leading and trailing edges of the plane airfoil along the lines $x = -1$ and $x = 1$ respectively, and assuming two-dimensional flow, the potential $\phi(x,t)$ at points along the chord is written as

$$\phi(x,t) = -\frac{1}{\beta}\int_{-1}^{x} G(x,\xi)\, w(\xi)\, e^{i\omega t}\, d\xi$$

where $\beta = (M^2 - 1)^{\frac{1}{2}}$, M is the Mach number of the free-stream; ω is the angular frequency; and $w(\xi)e^{i\omega t}$ is the velocity at points along the chord normal to the oscillating airfoil. The function $G(x,\xi)$, which plays the role of a Green's function, is shown to be given by

$$G(x,\xi) = e^{i\kappa M(x-\xi)} J_0[\kappa(x-\xi)]$$

where $J_0[\,]$ is the Bessel function of the zeroth order, $\kappa = kM/(M^2 - 1)$, and $k = \omega/U$ is the reduced frequency.

The lift and mid-chord moment produced by the plunging and pitching motion of the airfoil are derived from the chordwise distribution of pressure corresponding to the expression for the potential. These quantities are obtained as functions of M and k. The results are presented graphically for $M = 1.2$, 1.4, 1.6, and 2, and for $k = 0$ to 1.5. Robert C. F. Bartels, U.S.A.

157. Jonathan Winson, "Motion of an unarticulated helicopter blade," *J. aero. Sci.*, Sept. 1947, vol. 14, pp. 520–526.

The paper considers the bending oscillation of an unarticulated helicopter rotor blade (i.e., a rotor blade rigidly clamped to the rotor hub) perpendicular to the plane of rotation.

The differential equations for the first two natural modes of vibration of the blade are expressed in terms of the angular position of the blade in azimuth. In the derivation of these equations the applied air forces are determined by Glauert's steady flow wing theory, and the effects of damping are neglected. The solutions of the equations are approximated by the constant and first harmonic terms of their Fourier-series representation. The method is illustrated by solving a sample problem for the case of a representative blade under specific flight conditions.

Robert C. F. Bartels, U.S.A.

158. M. D. Haskind, "Oscillations of a wing in a subsonic flow" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Jan.–Feb. 1947, vol. 11, no. 1, pp. 129–146.

Utilizing the conventional linearization of the differential equations and boundary conditions pertaining to an oscillating two-dimensional profile in an otherwise undisturbed inviscid subsonic gas flow, the author obtains the Possio integral equation.

In order to avoid the necessity of solving this equation, the author reverts to the original (linearized) differential equation for the velocity potential, splits the solution into two additive parts, and using an elliptic co-ordinate transformation finds the first part as a product series of the odd Mathieu functions. For

the second part, he obtains both a closed integral expression and an even Mathieu function series.

Although no integral equation need now be solved, there is the difficulty of evaluating certain integrals and coefficients for the infinite series. In certain cases these are relatively simple but one must still sum the series. Although absolute convergence is claimed, no information seems available as to the number of terms required for the computation of useful quantities.

Some discussion is given to the downstream behavior which, of course, includes a vortex sheet. Lengthy discussion is also given to the energy balance, i.e., to the balance among expended mechanical energy, acoustic energy, energy associated with the vortex sheet, and the propulsion associated with the oscillation.

G. F. Carrier, U.S.A.

159. E. L. Leppert, Jr., "An application of IBM machines to the solution of the flutter determinant," *J. aero. Sci.*, Mar. 1947, vol. 14, pp. 171–174.

A method is indicated for the use of International Business Machines' punch card machines for performing the arithmetical operations required for solving the flutter determinant by a trial-and-error procedure. This method is particularly useful for investigating the effect of variation of parameters.

M. V. Barton, U.S.A.

160. A. P. Panichkin, "Forces acting on an oscillating profile in a supersonic gas flow," (in Russian) *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Jan.–Feb. 1947, vol. 11, no. 1, pp. 165–170.

The differential equations for the potential of a disturbance in a uniform supersonic stream are linearized in the conventional manner. Solutions which are harmonic in time are sought, to describe the disturbance associated with an oscillating thin two-dimensional profile. The boundary conditions at the profile are linearized, the time dependence "split off," and the resulting two-dimensional hyperbolic differential equation is readily solved by the Riemann method. In this case, the Riemann function is a zero-order Bessel function.

The dynamic lift and turning moment on the wing are evaluated for the particular cases where the wing (1) oscillates about the leading edge, (2) "flaps" in a direction normal to its chord. The results appear as series of Bessel functions.

G. F. Carrier, U.S.A.

161. A. R. Collar, "Aeroelastic problems at high speed," *J. roy. aero. Soc.*, Jan. 1947, vol. 51, no. 433, pp. 1–34.

This paper is primarily a descriptive survey of the wartime research of the author and his associates, with the major emphasis placed on the effects which airframe deformability has upon the stability and control characteristics. Flutter problems are also taken into consideration.

The loss of rolling power of a conventional wing-aileron combination at supersonic speeds is discussed, and the use of rotating wing tips or power-operated large-chord ailerons is suggested. It is shown that the effectiveness of the elevators may be greatly reduced at high speed by the bending of the fuselage and twisting of the tail plane (stabilizer), and that the operation of an aircraft near the elevator reversal speed is likely to cause a structural failure. It is also shown that the change in static longitudinal stability due to twisting of the tail plane is critically dependent upon the tail-plane setting.

The effect of bending upon the aerodynamic loading of a swept-back wing, and the consequent effects on the effectiveness of lateral control and on the longitudinal stability are discussed qualitatively. The destabilizing effect of wing bending during a pull-out is considered to be particularly significant. The paper includes an extensive bibliography.

M. J. Turner, U.S.A.

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 115, 117, 132, 137, 148, 157)

162. E. Stage, "Tests with reduced scale models for hydraulic turbines (Essais sur modèles réduits pour turbines hydrauliques)," *Rev. univ. Min.*, 1947, vol. 3, no. 5, pp. 191–198.

The general rules of similitude are briefly reviewed and their application to hydraulic turbines is discussed. The influence of the scale on the efficiency is indicated. The author points out that, although the laws of similitude indicate that the efficiency is improved in a full-sized turbine compared with that of its reduced model, one should not count on this, since the rules of similitude are not rigorous enough unless the model is exactly identical with the turbine to be designed.

Cavitation phenomenon must also be taken into account. For this purpose the suction head of the test turbine may be changed until cavitation occurs. Cavitation appears when the results obtained deviate from those calculated according to the rules of similitude. Diagrams of power, efficiency, and the coefficient of cavitation are given as functions of pressure head. Also diagrams of power, efficiency, and the flow are given as functions of the cavitation coefficient.

Ahmed D. Kafadar, U.S.A.

163. Werner T. von der Nuell, "The radial turbine, especially with regard to gaseous substances: Part I. The radial turbine with inward flow (centripetal turbine), and Part II. The radial turbine with outward flow (centrifugal turbine)," *ATI Tech. Data Dig.*, Sept. 1, 1947, vol. 12, no. 5, pp. 5–31.

In Part I of this paper, general formulas are derived on the basis of one-dimensional flow theory, interrelating adiabatic head, volume flow, efficiency, reaction, and nozzle vane angles. Influence of these various factors on the peripheral efficiency and the velocity ratio are shown in curves. Comparison of these estimated values with axial turbines shows potential advantages in certain regimes.

The author points out that a turbine with straight radial blades should afford a simple, light construction with estimated efficiency in the neighborhood of 90 per cent. Control of the characteristic curve by adjustment of nozzle vane angles is considered promising.

In Part II, the flow through a centrifugal turbine (outward flow) is shown to be inherently less advantageous than for the centripetal turbine (inward flow). However, it is demonstrated that a simple and compact two-stage turbine could be developed by utilizing a centripetal turbine for the first stage and centrifugal for the second stage. In general, it is concluded that the radial turbine may have specific advantageous applications which would warrant further theoretical and experimental investigations.

Frank L. Wattendorf, U.S.A.

Experimental Flow Equipment and Technique

(See also Revs. 4, 129, 193)

164. S. H. Hasinger and W. T. von der Nuell, "Momentum measurement by balancing an impact pendulum," *ATI Tech. Data Dig.*, July 15, 1947, vol. 12, no. 2, pp. 7–20.

Experiments have been completed on an impact pendulum device for measuring the thrust of a jet engine. Careful tests were made to prove that the entire momentum of a jet could be transferred to a plate directly in the path of the jet, so that the resulting thrust could be measured accurately. A pendulum plate design was developed having a network of wire on the impact face such that errors caused by induced flow around a simple plate were eliminated. The method was found to be closely accurate for jets under both pulsating and steady flow conditions.

The principles of design worked out by laboratory tests on a small scale are readily applicable to full-size engine tests. The Air Materiel Command is currently investigating the application of this method to the testing of jet engines, particularly in the field. E. G. Allen, U.S.A.

165. R. Buchdahl, J. G. Curado, and R. Braddicks, Jr., "A variable speed rotational viscosimeter," *Rev. sci. Instrum.*, Mar. 1947, vol. 18, no. 3, pp. 168–172.

This paper describes the design and operation of a precision type of viscosimeter using concentric rotating and fixed cylinders. Measurement of the torque is accomplished by means of an electric strain gage. The instrument is adapted to the measurement of viscosities in the range from 10^{-1} to 10^{4} poises, with an accuracy of ± 3 per cent. M. J. Thompson, U.S.A.

Hydraulics; Transport of Solids; Cavitation

(See also Revs. 122, 124, 162)

166. Duilio Citrini, "Recent investigations concerning the diffusion of a liquid vein in a static liquid field (Nuove ricerche sulla diffusione di una vena liquida in un campo di liquido in quiete)," *Energia elett.*, June 1947, vol. 24, no. 6, pp. 177–192.

This article, the third of a series which the author has published on the subject, describes tests designed to investigate the validity of his theoretical studies, and to supplement previous experimental results.

A submerged orifice, 34 mm in diam, was used to discharge water from a tank into an observation flume under differential heads between 30 and 85 cm. Both axial and transverse velocity profiles, obtained from Pitot-tube readings, are shown. The data are generalized, within the range of observation, by using dimensionless variables.

The process of diffusion of the jet into the surrounding static fluid appears to be independent of the head-diameter ratio. Good agreement is shown between the theory and the test results. Glenn Murphy, U.S.A.

167. Ralph Alger Bagnold, "Sand movement by waves: some small-scale experiments with sand of very low density," *J. Instn. civ. Engrs.*, Feb. 1947, vol. 27, no. 4, pp. 447–469.

This describes tests and analyses of the mechanism of sand movement by waves seaward of the "plunge point" on beaches. Existing bed-water movement theories are summarized, including movement caused directly by waves and currents due to other causes.

Theories of wave trains and solitary waves are compared, and the critical wave period is derived above which a train disintegrates into solitary waves. Bed-water movement is compared for periods above to below critical. For infinite surfaces the entire water body is shown to be moved forward by solitary waves, while a critical period train produces only symmetrical oscillations The two theories disagree on movement per wave, that of the solitary wave being greater. For finite two-dimensional channels this is modified by reverse current to give zero mean movement. For this case the analysis indicates: (1) reduced surface wave velocity; (2) horizontal bed-water movement changed from a series of forward displacements to an oscillation whose asymmetry increases with the period; (3) full horizontal amplitude with long periods, and 50 per cent reduction with near-critical periods.

The experiments were designed to test these conclusions for long-period waves, compare loose sand beds with the assumed smooth bed, and to study the amount of sand transportation. Test equipment was a horizontal glass-walled flume, with a wave-machine at one end and a sloping beach at the other. Water movement was observed with dye streaks.

With a smooth bed, wave velocities were lower than calculated, with higher reverse currents. A strong turbulent forward drift near the bed persisted up the rising beach to the plunge point, then rose and drifted back. Shoreward of the plunge point there was no outward drift. Sands A, B, and C were used, having apparent densities in water of 0.18, 0.18 and 1.65, and grain diameters of 0.05, 0.02, and 0.006 in. All showed the same qualitative behavior but differed in degree, with A showing the largest and C the smallest effects. Horizontal water amplitudes were below the "natural ripple-pitch" for A and B. Extensive observations on formation and drift of bars, change of wave and beach profile and of sand structure were made, covering prototype periods from one to several years.

Robert T. Knapp, U.S.A.

168. Charles Jaeger, "Underground water tables in the steady state (Remarques sur les nappes souterraines en regime permanent)," *Schweiz. Bauztg.*, Mar. 22, 1947, vol. 65, no. 12, pp. 152–155; and Mar. 29, 1947, no. 13, pp. 163–166.

The author treats the problem of underground water table as a particular case of free surface flow, taking into account the existence of a velocity potential. It is shown how Dupuit's theory (which assumes that velocities along the same vertical are all equal) is not accurate in general for calculating water table.

The paper discusses: (1) Underground water table considered as a case of "free surface flow"; (2) integration of free underground water table in steady state, flowing along a horizontal impermeable plane toward a well or trench; (3) comparison of the theoretical results with the results of tests or observations in nature; (4) problem of improving grounds by drains; (5) other types of flows (other problems than classical ones of wells, trenches, and drains in a homogeneous ground, limited underneath by a horizontal impermeable plane), including such practical problems as trenches with impermeable inclined bottoms, etc.

It is stated that methods described in the paper are applicable also to transient flow. Ahmed D. Kafadar, U.S.A.

169. E. E. Ambrosius and L. K. Spink, "Coefficients of discharge of sharp-edged concentric orifices in commercial 2-in., 3-in., and 4-in. pipes for low Reynolds numbers using flange taps," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, no. 8, pp. 805–812.

This paper reports on orifice coefficient investigations at the University of Oklahoma begun in 1933. Orifice coefficients are given over a range of Reynolds numbers from 40 to 10,000 and for Beta (orifice to pipe diameter) ratios up to 0.75. All tests were made in standard commercial pipe so as to be directly applicable to field installations. An attempt was made to correlate the data at the higher Reynolds numbers with coefficients previously published in Amer. Soc. mech. Engrs. Fluid Meters Reports.

The value of the paper is considerably enhanced by the discussions presented by Folsom and Peterson, the former in particular because of its consideration of the problem from the standpoint of basic fluid mechanics. The comments of Bean and Beitler regarding the issuance of standard values of orifice coefficients for this low Reynolds number range are also of interest. M. J. Thompson, U. S. A.

170. M. F. M. Osborne, "The shock produced by a collapsing cavity in water," *Trans. Amer. Soc. mech. Engrs.*, Apr. 1947 vol. 69, no. 3, pp. 253–266.

Collapse of vapor-filled cavities containing varying but known amounts of air was studied experimentally. The apparatus involved a small cylindrical vessel, one end of which was a plate-glass observation window. A magnetically released sylphon bel-

lows in the other end provided the volume change necessary for formation and collapse of the cavity. Deaerated water was used. Air bubbles were introduced and the volume measured at atmospheric pressure. The cavity was then increased by extending the bellows. Collapse was initiated by releasing the bellows. The resulting pressures were measured by a microphone inserted in the vessel.

To the ear the sound varied from a dull thud with large air content, to sharp clicks for low air content. The measured pressure rise increased as air content decreased. In all cases multiple shocks were observed, indicating pulsation, i.e., a series of regrowths and recollapses instead of a single collapse. An attempt was made to correlate the time of collapse with the prediction of the Rayleigh theory which assumes an incompressible fluid. The calculations are somewhat uncertain due to inertia of the moving parts of the apparatus. Discussions by M. P. O'Brien, T. C. Poulter, and B. G. Rightmire are included.

Robert T. Knapp, U.S.A.

171. Mario Viti, "Method for determining the profile of the free surface for steady flow in cylindrical stream beds (Metodi per il tracciamento del profilo di pelo libero delle correnti in moto permanente in alvei cilindrici)," *G. Gen. civ.*, Mar.–Apr. 1947, vol. 85, no. 3–4, pp. 132–142.

The author discusses the general problem of determining the free-surface profile of water flowing in a stream bed of constant cross section under the condition of steady flow. The difficulties encountered in the general solution are pointed out, and a review of the solutions presented by other investigators is included.

The method proposed by the author consists in replacing the general expression for $\phi(h)$ by a polynomial which may be made to pass through three selected points on the $\phi(h)$ curve, and to which standard integration procedures are applicable.

A series of tables is given to assist in the solution for various standard cross sections. The sequence of operations to be followed in a given problem is indicated in detail, and it is stated that the method, in addition to being rapid, gives results which appear to be acceptable in the majority of practical cases. It is adaptable whether the depth is greater or less than the critical depth, but it is restricted to open channels of constant cross section.

Glenn Murphy, U.S.A.

Lubrication; Wear

(See also Rev. 200)

172. Charles P. Boegli, "The hydrodynamic lubrication of finite sliders," *J. appl. Phys.*, May 1947, vol. 18, pp. 482–488.

The author presents a short discussion pertaining to the determination of the performance of a slider bearing having finite dimensions, i.e., length and width. The classical Reynolds equation is solved by introducing two assumptions: (1) That pressure functions along the length and width of the slider are independent; and (2) that the pressure function along the length of the slider is the same as that for an infinitely long slider. Equations thus obtained permit the determination of such variables as load, center of pressure, force, and quantity of lubricant for flat as well as curved sliders.

Examples indicate that accurate performance data may be obtained by the use of the equations presented, especially when the ratio of length to width is equal to unity or less. The temperature rise of the lubricating oil is not discussed.

L. M. Tichvinsky, U.S.A.

173. P. G. Forrester, "Frictional properties of oil-seal materials," *Engineering*, Aug. 8, 1947, vol. 164, no. 4254, pp. 121–124.

This paper describes tests on the coefficient of friction and wear between various sealing materials and a steel plate, under conditions of boundary friction. The seal material was bent over the rounded ends of three plugs sliding on a rotating base plate. "Pool" lubrication (with excess lubricant) and "film" lubrication were investigated with some standard lubricating oils. Speeds were between zero and 2.5 cm per sec.

At speeds above 0.1 cm per sec the coefficient of friction for leather remained fairly constant at about 0.1, with higher values at 100 C and higher velocities. For room temperature pool lubrication, and standard surface finish, synthetic rubber materials gave somewhat smaller coefficients of friction, but their friction properties are not as good at 100 C and with film lubrication. Conditions leading to increased wear were film lubrication, temperature of 100 C, machined surface finish, and radially ground surface finish.

Erle I. Shobert II, U.S.A.

174. Lucien Leloup, "Study of a regime of lubrication: film friction of journal bearings (Etude d'un régime de lubrification: Le frottement onctueux des paliers lisses)," *Rev. univ. Min.*, 1947, vol. 3, no. 10, pp. 373–419.

Bearing performance experiments are described in a rather explicit way. During these experiments transmission-type bearings operating primarily under semifluid lubrication were investigated.

The range of speeds varied between 0.63 and 133 rpm, and the pressures varied between 45 and 450 psi. Over a dozen different types of lubricating oils, such as mineral, animal, and vegetable, were used. In addition experiments were also carried on with such fluids as glycerin and syrup of sugar.

The results obtained are presented graphically on curves giving the relationship between the coefficient of friction and the bearing characteristic number. The article is well illustrated by 13 photographs of several testing arrangements and by a total of 34 graphs and sketches, 26 of which present test results.

L. M. Tichvinsky, U.S.A.

175. Ch. Hanocq, "Investigations into the law of heat dissipation in bearings cooled by natural convection (Recherches sur la loi de dissipation de la chaleur dans les paliers refroidis par convection naturelle)," *Rev. univ. Min.*, 1947, vol. 3, no. 7, pp. 245–258.

In this paper the author describes bearing heat dissipation tests. Small oil-ring lubricated bearings of 40 and 60 mm in diam, and operating at 1000 rpm under a load of 500 kg (giving approximately 70 to 45 psi pressure on projected areas) were studied. They were run until steady-state conditions were reached.

The heat dissipation was measured, and these data are recorded as effects of the bearing metal and of the lubricating oils, and the influence of over-all dimensions. The results are compared with those previously obtained when bearings of 160 and 190 mm in diam were tested. All the tests described were made in quiet air.

L. M. Tichvinsky, U.S.A.

176. M. C. Shaw, "An analysis of the parallel-surface thrust bearing," *Trans. Amer. Soc. mech. Engrs.*, May 1947, vol. 69, no. 44, pp. 381–387.

The pressure distribution, load capacity, and friction characteristics of a parallel-surface thrust bearing are analyzed for a lubricant of variable density, but of constant viscosity. Side leakage is neglected, and it is assumed that the density varies linearly with distance in the direction of motion.

The theory confirms the experimental result described in a paper of A. Fogg ["Fluid film lubrication of parallel thrust surfaces," *Engineering*, Feb. 16, 1945, vol. 159, pp. 138–140] that hydrodynamic lubrication of parallel-surface bearings is

possible. It does not, however, bear out quantitatively the fact that at high speeds (considerably higher than has been common for shafts subjected to high thrust loads) the parallel-surface bearing is capable of supporting loads comparable in magnitude to those carried by thrust bearings in which taper is possible.

Furthermore, it does not corroborate the linear relationship between the coefficient of friction and the dimensionless variable $\sqrt{\mu N/p}$ at high rotative speeds. It is concluded that although the theory appears adequate for low speeds, it must be extended to account for high speed results. F. J. Maginniss, U.S.A.

Dynamics of Meteorology and Oceanography

(See also Revs. 152, 167, 198)

177. **C. H. B. Priestley and W. C. Swinbank, "Vertical transport of heat by turbulence in the atmosphere,"** *Proc. roy. Soc., London, Ser. A*, June 3, 1947, vol. 189, no. 1019, pp. 543–561.

The classical theory of turbulent heat transfer in the atmosphere shows that the flux of heat is from regions of high potential temperature; thus the heat transfer must be downward in a stable atmosphere. The authors quote experimental data indicating that the heat transfer in some conditions is very much less than that given by the classical theory if an exchange coefficient based on momentum transfer is used. A physical mechanism which may explain this well-known need for using different exchange coefficients for heat and for momentum is presented.

It is first noted that experimentally determined fluctuations of temperature at a point in the atmosphere may be much larger than fluctuations predicted by the classical mixing length theory, if mixing lengths are used which are suitable for momentum transfer considerations. This indicates that a given eddy may perform a considerable number of vertical oscillations and be carried a considerable distance from its level of origin before its thermal identity is lost. Any eddy which is far from its thermal equilibrium level must be acted on by a buoyant force, and this force, which is neglected in the classical theory, produces an upward transfer of heat. The authors thus write the energy flux upward as the sum of the classical expression, involving an essentially positive exchange coefficient, and a second buoyancy term involving the vertical turbulent velocity component and the turbulent component of the temperature.

The authors point out that this energy flux expression may be reduced to the classical form by introducing a "buoyancy length." However, the corrected exchange coefficient may be much less than the momentum exchange coefficient for a stable atmosphere. The authors suggest that the exchange coefficient might even be negative, but this appears to the reviewer to be inconsistent with the suggested mechanism. H. J. Stewart, U.S.A.

178. **G. E. R. Deacon, "Relations between sea waves and microseisms,"** *Nature, London*, Sept. 27, 1947, vol. 160, no. 4065, pp. 419–421.

The applications of the relation between sea waves and microseisms are shown to be of potentially great importance in studying atmospheric disturbances. The author at the same time makes clear the possible theoretical implications regarding standing waves on the ocean surface, which are being revealed by simultaneous microseismic and sea-wave data. These data indicate: (1) That the dominant microseismic disturbances originate over oceans; and (2) that they are usually generated over regions where standing waves are produced, by reflection from a coast or in a storm center where interference of several wave trains is likely. These observations are in accordance with other findings which showed that a steady monsoon wind did not produce microseisms.

Although it is a well-known result that the pressure oscillations in moving surface waves die out exponentially with depth, the author gives reference to new theoretical work which indicates that in standing waves the pressure variation extends to the sea bottom, producing there an oscillation of twice the frequency of the moving waves. This result fits strikingly well the observations made in two separate places that microseismic periods tend to be approximately one half that of the generating wave periods.

Joanne G. Starr, U.S.A.

179. **J. J. Stoker, "Surface waves in water of variable depth,"** *Quart. appl. Math.*, Apr. 1947, vol. 5, pp. 1–54.

The paper presents some of the results obtained at the Institute for Mathematics and Mechanics of New York University. The author investigates the wave lengths, amplitudes, and plane velocities of waves approaching a shore having bottom slopes of 90 deg (a vertical cliff), 45 deg, and 6 deg. The problem is solved in terms of analytic functions of a complex variable, using tables recently developed. The results are compared with those obtained by the shallow water theory which until this time has been applied to the problem.

The reflection from the shore is considered to be zero, an assumption which, in the case of small slopes, is stated to be justified by experimental determination of plane velocities. The waves are considered to be of small amplitude and the velocities of the particles sufficiently small so that the nonlinear terms characterizing the free surface can be dropped. The familiar breakers are therefore not covered.

The paper will be of primary interest to mathematicians as a useful approach to a complicated problem.

F. Everett Reed, U.S.A.

Ballistics

180. **A. C. Charters, "Some ballistic contributions to aerodynamics,"** *J. aero. Sci.*, Mar. 1947, vol. 14, pp. 155–166.

This paper discusses the results of a systematic series of investigations concerning the drag of various-shaped projectiles or aerodynamic bodies. Theoretical studies of the factors influencing the drag coefficients of the component parts of a projectile body were made, and total drag coefficients were obtained experimentally in the spark photography range of the Ballistics Laboratory, Aberdeen Proving Ground. Head shapes were analyzed by the von Kármán-Moore and Taylor-Maccoll theories. Compilation of drag data at different Mach numbers has resulted in simple empirical formulas which give a good approximation of the variation of drag coefficient with Mach number. Effects of boattailing, banding, scale, skin friction, and interaction of head on base are discussed. Firing tests were made at Mach numbers up to about 2.5.

The studies and ballistic test results are applied to design considerations of a supersonic aircraft fuselage shape, giving minimum drag with a fixed body diameter, and with assumed conical head and conical boattail. Frank L. Wattendorf, U.S.A.

181. **H. Jones, "A theory of the dependence of the rate of detonation of solid explosives on the diameter of the charge,"** *Proc. roy. Soc., London, Ser. A.*, May 1, 1947, vol. 189, no. 1018, pp. 415–426.

This deals with the reduction of detonation velocity in a cylindrical charge of solid explosive by radial expansion within the reaction zone. The author assumes that the flow in the reaction zone is one-dimensional, that the Chapman-Jouguet condition applies at the end of the reaction zone where the reaction is complete, and that the internal energy is proportional to the temperature. The equation of state is corrected by introducing the condition of constant covolume. He is then able to

derive an approximate relation between the velocity of a detonation wave and the relative radius at the end of the reaction zone.

To evaluate the radius of the stream tube in terms of that of the charge, two cases are considered: (*a*) the radial expansion of a bare cylindrical charge, and (*b*) the radial expansion of a cased cylindrical charge. The first problem considers the charge detonating in a vacuum. The flow pattern of the jet is approximated by that generated by rotating a streamline of the Prandtl-Meyer flow about the axis of symmetry, on the condition that the deviation of pressure so obtained from that corresponding to one-dimensional nozzle flow is a minimum. The second case considers a special problem of the charge surrounded by a metal case.

Y. H. Kuo, U.S.A.

182. J. McKeown, "Investigations on delay-action fuses," *Engineering*, Aug. 1, 1947, vol. 164, no. 4253, pp. 97–101.

This paper reports the work of the British Non-Ferrous Metals Research Association in the development of a type of delay-acting fuse mechanism. The mechanism was to operate on the principle of the creep of lead or lead alloys under the action of stress, the delay time being a function of creep rate. The final design consisted of a reduced section lead tension member, spring-loaded to cause creep and eventual fracture, thereupon permitting the striking of a percussion cap setting off the fuse.

The paper gives tension and torsion creep data at various temperatures for the lead and lead alloys used. Reliability as to timing for a particular fuse metal under fixed testing conditions was found to depend upon uniformity of grain size and of previous cold work.

Louis F. Coffin, Jr., U.S.A.

183. J. Corner, "The internal ballistics of a leaking gun," *Proc. roy. Soc., London, Ser. A*, Jan. 30, 1947, vol. 188, no. 1013, pp. 237–255.

The author here treats situations in interior ballistics for which the propellant gases can escape to the atmosphere prior to the exit of the projectile from the muzzle of the gun. Such questions are particularly pertinent where (1) Vents at the breech are introduced by the designer to cut down the recoil of the weapon; (2) barrel wear or variations in the manufacture of projectiles lead to gas leaks past the projectile while still in the barrel.

Treating the leakage problem as one of adiabatic flow through a nozzle, the author sets up equations which define approximately the variation of temperature and pressure of the propellant gases as the projectile moves down the barrel. If one assumes that the rate of burning of the powder grains is directly proportional to the first power of the pressure and that the flow is through a nozzle of constant throat area throughout the motion, approximate solutions of these equations can be found. Where the rate of burning is proportional to some power of the pressure and the nozzle throat area varies in time, numerical integration must be used.

In the main, guns with gas leaks behave like guns without leaks, but with a reduced charge, and lowered gas temperatures, gas pressures, and muzzle velocities. It also seems that after the propellant is completely burned, the pressure-travel curve falls off much more rapidly than it does in a gun not subject to gas leakage.

Benjamin Epstein, U.S.A.

184. J. Corner, "The internal ballistics of a gun after shot ejection," *Proc. roy. Soc., London, Ser. A*, Jan. 30, 1947, vol. 188, no. 1013, pp. 255–272.

The motion of the gas in a gun after the shot has left the gun is studied in this paper. It has long been recognized that theoretical information of this sort is needed in order to properly design recoil mechanisms and muzzle brakes. The problem of emptying a reservoir of perfect gas by expansion through a nozzle was first treated by Hugoniot in 1886. In 1919 and 1932 Rateau improved the earlier results by (1) including the effect of the covolume, and (2) correcting for the initial distribution of gas velocities along the barrel at the moment of shot ejection.

In most cases the results of Rateau and Hugoniot are sufficiently accurate for design purposes. However, by simple modifications of the Hugoniot-Rateau theory, the present author succeeds in getting somewhat better agreement between the predictions of theory and actual breech pressure-time curves. Considerable attention is paid in the latter part of the paper to the effects caused by the rarefaction wave set up in the gun when the shot leaves the muzzle. This effect is of importance in muzzle-brake calculations. The results of this paper are applicable where the muzzle velocities are in the neighborhood of 3000 ft per sec.

Benjamin Epstein, U.S.A.

Thermodynamics

(See also Revs. 132, 137)

185. Gustave Ribaud, "Influence of dissociation on the adiabatic expansion of a gas mixture at high temperature (Influence de la dissociation sur l'évolution adiabatique d'un mélange gazeux à haute temperature)," *C.R. Acad. Sci., Paris*, July–Dec. 1947, vol. 225, pp. 363–366.

A discussion of a paper by M. Serruys [*C. R. Acad. Sci., Paris*, 1946, vol. 223, p. 980] dealing with the evaluation of the coefficient of polytropic expansion in the presence of chemical dissociation is presented.

For a system fully isolated from the surroundings, considering only the heat of chemical reaction, he derives the expression

$$K = \frac{C_p - \left(\frac{dQ}{dT}\right)_p + \bar{R}T\left(\frac{dn}{dT}\right)_p}{C_v - \left(\frac{dQ}{dT}\right)_p} \quad \ldots\ldots\ldots [1]$$

where K = coefficient of polytropic expansion; C_p, C_v = specific heats at constant pressure, volume; n = moles of the gas; $\bar{R}$ = universal gas constant; p,v,T = pressure, volume, and temperature; and Q = constant pressure heat of reaction. If some slight approximations are made Equation [1] can be reduced to

$$K = \frac{C_p - \left(\frac{dQ}{dT}\right)_v}{C_v - \left(\frac{dQ}{dT}\right)_p} \quad \ldots\ldots\ldots [2]$$

For the mixture $CO_2 + 2N_2$, at 10 atm and 2500 C, relation [1] gives for K the value of 1.13; on the other hand if the influence of dissociation is neglected the value of $K = c_p/c_v$ would have been 1.24. In discussing the effect of dissociation further the author concludes that it is not possible to extend the relations used for inert gases to cases where adiabatic expansion is accompanied by chemical dissociation. Y. S. Touloukian, U.S.A.

186. J. H. Keenan and Joseph Kaye, "A survey of the calculated efficiencies of jet power plants," *J. aero. Sci.*, Aug. 1947, vol. 14, pp. 437–450.

Performances of different aircraft power plants are compared on the basis of an efficiency defined as propulsive thrust times distance moved, divided by heat supplied. Curves presented show the effects of various combinations of variables, including aircraft speed, turbine inlet temperature, turbine and compressor efficiencies, reheat, and regeneration. Methods of calculation are indicated, but details are not given.

Results are applicable to operation at 30,000 ft (9100 m) altitude, and are mostly comparative rather than absolute, since reversible processes and zero pressure losses are assumed. For

speeds less than 600 mph (970 km per hr), the order of increasing efficiency is: ramjet, regenerative ramjet, turbojet, turbojet with reheat, turbopropeller, regenerative turbopropeller, the same with reheat, compound engine. For supersonic speeds the turbojet and ramjet are best, the former being more efficient up to 1700 mph (2740 km per hr). C. W. Smith, U.S.A.

187. H. Desmond Carter, "The loop scavenge Diesel engine," *Proc. Instn. mech. Engrs.*, 1946, vol. 154, no. 4, pp. 386–411.

A discussion is given of the two-cycle loop scavenge Diesel engine. The advantages of this engine in the way of simplicity, high power at low speeds, easy reversing, etc., are enumerated, and comparisons are made with poppet-valve engines, two and four cycle. The design problems in obtaining competitive efficiency and full capacity are discussed.

Particular reference is made to a new method, tested by the author, of charging the engine cylinder, called "exhaust pulse pressure charging," in which the pressure pulse from the exhaust of one cylinder is used to increase the charge of a second cylinder. Data are given for the performance of test engines.

Stanley Thompson, U.S.A.

188. Eddy Mezger, "Empirical relation between the density of a liquid and the density of saturated vapor (Relation empirique reliant la densité d'un liquide à la densité de la vapeur saturée)," *C. R. Acad. Sci., Paris*, July 16, 1947, vol. 225, pp. 108–109.

The author finds that the relation between the densities of saturated liquid d_l and saturated vapor d_g can be represented for all temperatures by the simple equation

$$d_l^{2/3} + d_g^{2/3} = 2d_c^{2/3}$$

where d_c is the critical density. This equation is stated to be accurate within ± 6 per cent for various organic and inorganic substances examined, and to give better results than the rule of rectilinear diameter by Cailletet and Mathias.

E. F. Lype, U.S.A.

189. E. Bodea, "Temperature and entropy in dimensionless units (Temperatur und entropie in dimensionskoharenten einheiten)," *Schweiz. Arch.*, Feb. 1947, vol. 13, no. 2, pp. 33–45.

The dimensional characteristics of thermodynamic quantities, in particular temperature, specific heat, and entropy, are discussed with the goal of reducing the number of independent dimensions and at the same time attaining a more fundamental interpretation of these thermodynamic quantities.

The principles of the kinetic theory of gases are used as a guide to define temperature in terms of molecular energy so that specific heat becomes a dimensionless quantity, and in particular the constant volume specific heat of an ideal monatomic gas becomes equal to one. The relation between entropy and the probability functions of statistical mechanics is emphasized. No new correlations between temperature and energy are introduced, and those recommended are selected arbitrarily.

Newman A. Hall, U.S.A.

190. L. B. Edelman, "The pulsating jet engine—its evolution and future prospects," *SAE Quart. Trans.*, Apr. 1947, vol. 1, no. 2, pp. 204–216.

This paper describes briefly and ably the status of the pulsating jet engine, beginning with its early evolution and including some historical data. The problems of instrumentation, fuel introduction, and theory are discussed, as well as engine geometry, air valves, and fuel studies. Progress achieved by analyzing operating frequencies, valve mechanisms, and timing as well as the results of pressure measurements and flame propagation are described, and present performance reported.

A discussion of future prospects of these engines necessarily expresses the personal opinion of the author, but some of the predictions for future applications will undoubtedly come true. It is noteworthy that some nonaircraft applications, such as a blowerless heating plant, are mentioned as a future field of application. H. E. Sheets, U.S.A.

191. Hans Reissner, "Systematic analysis of thermal turbojet propulsion," *J. aero. Sci.*, Apr. 1947, vol. 14, pp. 197–210.

The author considers the performance characteristics of the turbojet engine, applying conventional thermodynamic and fluid dynamic principles to indicate a procedure for step-by-step determination of efficiency and output. All internal elements are assumed to have characteristics described by suitable parameters. The exposition is highly analytical and contains many unusual dimensionless parameters and much new notation.

Newman A. Hall, U.S.A.

192. Stefan Procopiu, "Interesting relation between the molecular heat of fusion and the temperature of fusion of chemical compounds (Valeurs remarquables du rapport entre la chaleur moléculaire de fusion et al température de fusion des composés chimiques)," *C. R. Acad. Sci., Paris*, Jan. 27, 1947, vol. 224, pp. 264–266.

Examination of the molecular heat of fusion gives the following empirical relation between temperature T and heat of fusion H_f: (1) for monatomic metals $H_f = RT$ (R = gas constant); (2) for 45 polyatomic organic and inorganic substances, including the derivatives of benzene in the ortho-position, $H_f = nRT/2$ (n = number of atoms in the molecule); (3) for the derivatives of benzene in the para-position $H_f = (n + 1)RT/2$. Certain alcohols, bases, and organic acids, presumably due to association in the liquid state, do not follow these rules, which should probably be applied only to those compounds verified in the paper.

E. F. Lype, U.S.A.

Heat Transfer

(See also Revs. 175, 177)

193. W. M. Rohsenow and J. P. Hunsaker, "Determination of the thermal correction for a single-shielded thermocouple," *Trans. Amer. Soc. mech. Engrs.*, Aug. 1947, vol. 69, no. 6, pp. 699–704.

A graphical method is presented for correcting the reading of a thermocouple in a hot gas stream, having a single radiation shield, for loss of heat to a wall colder than the gas stream. The graphs establishing this correction are obtained by solving for the unknown gas temperature from two simultaneous equations expressing the energy balances of the thermocouple tip and of the shield. Stanley Thompson, U.S.A.

194. T. A. Hall and P. H. Tsao, "Heat transfer at low temperatures between tube walls and gases in turbulent flow," *Proc. roy. Soc., London, Ser.* A, Sept. 26, 1947, vol. 191, no. 1024, pp. 6–21.

This paper describes an apparatus designed on the counterflow system, to study heat transfer between tube walls and gases, at low temperatures in a egion in which careful measurements had not previously been made. Oxygen, nitrogen, and carbon dioxide are studied, with a temperature range from +45 to −167 C, pressures up to 11 atm, and Reynolds numbers from 3000 to 60,000.

Results were correlated by the use of dimensionless groups, and a general equation obtained, applicable over the range of the experiments. With Reynolds numbers evaluated at mean film temperatures, the coefficient in the equation was found to be 5

per cent lower than that obtained from measurements made at normal and high temperatures. This is regarded as justifying the extension of the ordinary equation to low-temperature regions, until further data are available regarding specific heat, thermal conductivity, and viscosity of the gases at these low temperatures.

Determinations on friction accompanying heat transfer with gases in turbulent flow at low temperatures showed that the effect of heat transfer on friction factor was small. A comparison shows the measured values of friction factor to be in fair agreement with those determined from the Reynolds analogy. Comparison with the more refined analogies is not considered.

Joseph H. Keenan, U.S.A.

195. M. Jakob, "Some investigations in the field of heat transfer," *Proc. phys. Soc.*, Sept. 1947, vol. 59, part 5, no. 335, pp. 726–755.

This paper presents many topics in radiation, conduction, and convection selected from the author's contributions in heat transfer. Experiments, with accompanying theory, are described adequately, but the original papers in the Bibliography should be consulted for details.

The article contains the following correlations of new data for heat transfer by free convection from a vertical cylinder to non-boiling liquids. The cylinder diameter is used as the characteristic length.

Laminar: $N_{Nu} = 0.726[N_{Gr}N_{Pr}]^{1/4}$, when $2 \times 10^8 \leqq N_{Gr}N_{Pr} \leqq 5 \times 10^{10}$

Turbulent: $N_{Nu} = 0.0674[N_{Gr}(N_{Pr})^{1.29}]^{1/3}$, when $6 \times 10^{10} \leqq N_{Gr}(N_{Pr})^{1.29} \leqq 3 \times 10^{12}$

where N_{Nu}, N_{Gr}, N_{Pr} are the Nusselt, Grashof, and Prandtl numbers. The first equation gives values 12 per cent higher, and the second gives values 15 to 40 per cent lower than would be obtained, according to estimates by the reviewer, from earlier sources.

Previously unpublished data are given for heat transfer from a circular plate perpendicular, or inclined, to an air stream. Contrary to previous opinion, heat-transfer coefficients under such circumstances are shown to approximate those for similar plates parallel to the stream. H. G. Elrod, Jr., U.S.A.

196. A. M. Whistler, "Correction for heat conduction through longitudinal baffle of heat exchanger," *Trans. Amer. Soc. mech. Engrs.*, Aug. 1947, vol. 69, no. 6, pp. 683–685.

As the author states: "Heat exchangers with two passes in the shell side have a longitudinal baffle in the center. Heat flowing through this baffle renders the exchanger less efficient than the ideal. A correction factor F is worked out whereby the ideal formula may be used:

$$\text{Surface} = \frac{\text{Heat}}{F \times \log(\text{mean temp. difference}) \times (\text{heat-transfer coefficient})}$$

The correction is for heat leakage only. The factor does not correct for leakage of fluid from one pass to the other."

Y. S. Touloukian, U.S.A.

197. Carl F. Kayan, "Temperatures and heat flow for a concrete slab with imbedded pipes," *Refrig. Engng.*, Aug. 1947, vol. 54, no. 2, pp. 143–151.

This paper presents a further set of examples of the use of the electrical analogy in heat-transfer problems, previously discussed by the author, in which the potential distribution in a conducting plate of a shape similar to the structure under consideration is used to determine the heat-flow characteristics of the structure.

In the present application of this method, the problem of pipes imbedded in concrete is investigated and the results are determined in terms of such parameters as pipe spacing, concrete thickness, and the surface conductance factors for both ceiling and floor. The tabulation of the results shows the variation of floor and ceiling temperatures with different pipe spacings and different surface conductances, and the variation of slab heat flow with tube spacing. An alternative method of calculating the results for the same problem is also presented.

Erle I. Shobert, II, U.S.A.

198. Hippolyte Parodi, "An approximate relationship between the amplitude of the daily temperature fluctuations and the daily terrestrial radiation upon a horizontal surface (Sur une relation approximative entre l'amplitude diurne de variation de température et la radiation journalière globale sur une surface horizontale)," *C. R. Acad. Sci., Paris*, July 7, 1947, vol. 225, pp. 48–50.

It has been observed that the amplitude of daily temperature variation in heated railroad cars was not proportional to the heat input, but followed the law

$$\Delta T = k\int \sqrt{W}\, dt$$

(W = instantaneous heat supply, k = constant). Subsequently, the question was examined whether such a law also applies to the heating of air by the heat of the sun, as stored in and reflected by the soil. Taking mean values of W for different hours of the day and integrating over periods of one month, gave a correlation with the observed maximum temperature fluctuations in which k was nearly constant for all months of a normal year.

E. F. Lype, U.S.A.

199. Philippe Marmet, "Calculation of damping indices and thermal lag of walls (Le calcul des indices d'amortissement et de promptitude thermique des murs)," *C. R. Acad. Sci., Paris*, Mar. 10, 1947, vol. 225, pp. 729–731.

The differential equations of heat conduction are written so that they are analogous to the equations of conduction of electricity in a conductor without self-induction, the heat flow taking the part of the current, and the temperature the part of the voltage. The linear heat flow through a body consisting of various layers is then equivalent to the flow of a current through various conductors with different values of impedance and conductivity. Using this analogy, the temperature field in the body is a vector field whose magnitude is the damping index and whose angle is the thermal lag, both of which can be calculated from the generalized law of Ohm the classical law of electric fields, and Kennelly's law of reflections. E. F. Lype, U.S.A.

200. S. A. Schaaf, "On the superposition of a heat source and contact resistance," *Quart. appl. Math.*, Apr. 1947, vol. 5, pp. 107–111.

As two solid bodies are rubbed together, work is done at the rubbing surfaces. This work raises the temperature of the rubbing surfaces and the energy is then transferred by conduction into the body of the solids. This process is idealized as a plane heat source plus surface resistance to heat flow, plus semi-infinite conducting media.

It is shown that for several arrangements of heat source and surface resistance, a single boundary condition applies. The appropriate differential equations are set up and solved, making use of Laplace transform methods. General solutions are obtained for the case of constant heat source (as for constant rubbing velocity) and for heat source which is a function of time (as for accelerated rubbing velocity, a projectile in a gun). A few sample computed temperature distributions for a copper block rubbing on steel are shown. Howard W. Emmons, U.S.A.